

## A Dispersion Analysis for Difference Schemes: Tables of Generalized Airy Functions\*

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**Abstract.** This paper contains graphs and tables of the function

$$Ai_{p,q}(\alpha, x) = \int_{-\infty}^{\infty} (2\pi)^{-1} \exp\{iy^p/p - \alpha y^q/q + ixy\} dy$$

and its indefinite integral for  $p = 3, 5, 7$ , for  $q = 2, 4, 6$ , and for several values of  $\alpha$  with  $\alpha \geq 0$ . It is shown how these tables should influence the choice of an artificial viscosity for a difference scheme for a linear hyperbolic equation.

**1. Introduction.** This paper consists primarily of graphs and tables of values of the generalized Airy function

$$(1.1) \quad Ai_{p,q}(\alpha, x) = (2\pi)^{-1} \int_{-\infty}^{\infty} \exp\{iy^p/p - \alpha y^q/q + ixy\} dy$$

and its integral

$$(1.2) \quad Aii_{p,q}(\alpha, x) = (2\pi i)^{-1} \int_{-\infty}^{\infty} y^{-1} \exp\{iy^p/p - \alpha y^q/q + ixy\} dy.$$

Here,  $\alpha \geq 0$ ,  $x$  is real, and  $p$  and  $q$  are natural numbers such that  $q$  is even and  $p$  is odd,  $p \geq 3$ . The improper integral (1.2) is the Cauchy principal value. We computed these tables because the solution of a linear hyperbolic equation with constant coefficients behaves near a discontinuity like the integral  $Aii_{p,q}$ . See [4] and [5]. Our tables show the magnitude and extent of the numerically induced oscillations.

Our results show that it is better to add an artificial viscosity to a nondissipative difference scheme than to use a difference scheme with the viscosity built in. The reason is that if  $q < p$  as in the upstream-difference method, the viscosity becomes predominant as  $t/h$  increases, and if  $q > p$  as in the Lax-Wendroff method, the viscosity weakens as  $t/h$  increases. On the other hand, for a nondissipative scheme with order of accuracy  $p - 1$ , if the artificial viscosity is a discretization of

$$(1.3) \quad Q = -(-1)^{q/2} \delta h^{q(p-1)/p} q^{-1} \partial^q u / \partial x^q$$

( $q$  even), then the amount of viscosity as measured by the parameter  $\alpha$  in (1.1) and (1.2) is independent of  $h$ ,

$$\alpha = \delta t^{(p-q)/p}.$$

We see that  $\alpha$  gives a measure of the effective diffusive time scale. Note, though, that with the artificial viscosity (1.3)  $\alpha$  is an increasing function of  $t$  if  $p > q$  and a decreasing function of  $t$  if  $p < q$ .

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Received September 30, 1976; revised January 23, 1978.

AMS (MOS) subject classifications (1970). Primary 65A05; Secondary 33A70, 65M15.

\*This work was performed under the auspices of the U.S. Energy Research and Development Administration under contract number W-7405-Eng-48.

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The behavior of a high-order difference scheme near a discontinuity is as follows. Suppose we use a nondissipative difference scheme with order of accuracy  $p - 1$  and artificial viscosity (1.3). Then for fixed  $t$  the oscillations near a discontinuity extend over a distance

$$O(h^{(p-1)/p}).$$

We see that there is a slight improvement as  $p$  increases. Our tables show that as  $p$  increases ( $p = 3, 5, 7$ ) there is also a decrease in the amplitude of the oscillations. As we increase  $q$ , though, the damping near the wave front is decreased. In fact, it is only for  $q = 2$  that there is no overshoot.

Let us emphasize that we consider here only linear equations with linear artificial viscosity. The behavior of difference schemes for nonlinear equations with shocks is different. Nonlinear artificial viscosities of the form

$$(1.4) \quad Q = -(-1)^m \delta h^{2m(p-1)/p} (2m)^{-1} D^m (|Du|^r D^m u)$$

are in common use [9, p. 313], [7], where  $Du$  denotes  $\partial u/\partial x$ . It is not usual, though, to use this particular power of  $h$ . The reason for using a nonlinear viscosity of the form (1.4) is that one wants the effect to be greatest when  $|Du|$  is large. If  $r$  is even, the computation is more efficient because we do not need to check  $\text{sgn}(Du)$ . Finally, let us remark that the clipping algorithm of Boris and Book [1] is very good for discontinuities of the sort  $u = \text{sgn}(x)$ , but for  $u = \text{sgn}(x) - x$  it produces a staircase.

We have concentrated on the behavior of a difference scheme near a discontinuity of the solution of the differential equation. It is also of interest to know the behavior near a discontinuity of a derivative. If the discontinuity were in the derivative of order  $r$ , we would need to compute tables of the  $(r + 1)$ -fold indefinite integral of the generalized Airy function  $Ai_{p,q}$ .

Our graphs and tables are in the Appendix which appears in the microfiche section of this issue. In Section 2 we describe the graphs and tables and tell how we computed them. In Section 3 we show how the generalized Airy functions apply to various difference schemes.

**2. Description of the Tables.** In the Appendix are three sets of graphs with adjoining tables. The first set (Figures 1–17 and Tables 1–9) gives four local extrema of  $Aii_{p,q}(\alpha, x)$ . The second set (Figures 18–23 and Tables 10–12) shows the function

$$(2.1) \quad F_q(x) = (2\pi)^{-1} \int_{-\infty}^{\infty} \exp\{-y^q/q + ixy\} dy$$

and its integral. Finally, the third set (Figures 24–105 and Tables 13–53) consists of graphs and tables for the functions  $Ai_{p,q}$  and  $Aii_{p,q}$ .

The function  $Aii_{p,q}(\alpha, x)$  is known [5] to oscillate about the value  $-1/2$  for negative values of  $x$ . If  $p = 3$  and either  $\alpha = 0$  or  $q = 2$ , we have

$$\lim_{x \rightarrow \infty} Aii_{p,q}(\alpha, x) = 1/2$$

monotonically. Otherwise, the function  $Aii_{p,q}(\alpha, x)$  oscillates about the value  $1/2$  for positive values of  $x$ . The amplitudes of these oscillations are important in numerical analysis. The first set of graphs and tables shows the values for  $p = 3, 5, 7$  and  $q = 2, 4, 6$  of  $Aii_{p,q}(\alpha, x)$  at the first three negative local extrema and at the first positive local maximum. Of the three negative local extrema the first is a local minimum and usually the absolute minimum, the second is a local maximum, and the third is a local minimum.

We point out some peculiarities. For  $q = 2$  and  $p = 5$  or  $p = 7$  the first two negative local extrema coalesce and disappear near  $\alpha = 3$  or  $\alpha = 3.2$ , respectively. In fact, we see values of  $\alpha$  for which the first negative local minimum is greater than  $-1/2$ . Similarly, for  $q = 2$  and  $p = 5$  the third and fourth negative extrema coalesce near  $\alpha = 4.8$ , and for  $q = 2$  and  $p = 7$  the first and second positive local extrema coalesce near  $\alpha = 4.1$ . For  $p = 3$  and  $q = 2$  there is no positive local maximum. For  $p = 3, q = 4$ , and  $\alpha < 0.3$  the first positive local maximum is too flat to compute accurately. For  $p = 3$  and  $q = 6$  the first two positive local extrema coalesce as  $\alpha$  decreases to some value between  $0.2$  and  $0.3$ .

The second set of graphs and tables is labeled "P-term missing" and shows the function  $F_q(x)$  defined by Eq. (2.1) and its integral, the Cauchy principal value of

$$Fi_q(x) = (2\pi i)^{-1} \int_{-\infty}^{\infty} y^{-1} \exp\{-y^q/q + ixy\} dy.$$

The importance of these functions is that they show the limiting behavior of the generalized Airy functions for large viscosity,

$$(2.2) \quad \lim_{\alpha \rightarrow \infty} \alpha^{1/q} Ai_{p,q}(\alpha, \alpha^{1/q}x) = F_q(x), \quad \lim_{\alpha \rightarrow \infty} Aii_{p,q}(\alpha, \alpha^{1/q}x) = Fi_q(x).$$

For this reason we include the case  $q = 2$ , even though  $F_2$  is the Gaussian function

$$F_2(x) = (2\pi)^{-1/2} \exp\{-x^2/2\},$$

and  $Fi_2(x)$  differs from the error function by  $1/2$ . It is clear that  $F_q$  is an even function, so that it would have been sufficient to plot and print its values for positive  $x$ .

For ease of comparison with the generalized Airy functions we used for both  $F_q$  and  $Aii_{p,q}$  a format appropriate for the Airy functions. We remark that since  $\exp\{-y^q/q\}$  and  $e^{ixy}$  are  $G$ -functions [8], it follows that  $F_q(x)$  is also a  $G$ -function. In fact, it is not hard to show that in terms of the hypergeometric function  ${}_0F_{q-2}$  we have

$$F_q(x) = \sum_{m=0}^{q-2} b_m x^m {}_0F_{q-2}(\ ; \delta_{m1}, \delta_{m2}, \dots, \delta_{m,q-2}; (-1)^{q/2} x^q / q^{q-1}),$$

$$b_m = 0 \quad (m \text{ odd}),$$

$$b_m = (-1)^{m/2} q^{(m+1)/q} \Gamma((m+1)/q) / (m! q \pi) \quad (m \text{ even}).$$

Here,  $\delta_{m1}, \delta_{m2}, \dots, \delta_{m,q-2}$  are the consecutive numbers  $(m+2)/q, (m+3)/q, \dots, (m+q)/q$  with  $q/q$  omitted.

The first three negative local extrema for  $Fi_4$  and  $Fi_6$  are as follows

$x$	$Fi_4(x)$
-2.44196790	-0.55220867
-4.79724416	-0.49384599
-6.81358116	-0.50082421
$x$	$Fi_6(x)$
-2.50081385	-0.57059517
-4.93258336	-0.47901803
-7.23239945	-0.50617746

Hence, by (2.2) we have the limiting behavior of the first three local extrema of  $Aii_{p,q}$  for  $q = 4, 6$ . It is obvious that  $Fi_2$  is monotone.

The final set of graphs and tables is of values of  $Ai_{p,q}(\alpha, x)$  and  $Aii_{p,q}(\alpha, x)$  for  $p = 3, 5, 7$  and  $q = 2, 4, 6$ . They appear in the order  $(p, q) = (3, 2), (3, 4), (3, 6), (5, 2), \dots$ . For all values of  $p$  and  $q$  we give the generalized Airy function and its integral for  $\alpha = 0.5, 1, 2, 4$ . The case  $\alpha = 0$  is given only for  $q = 2$ , because  $Ai_{p,q}(0, x)$  is independent of  $q$ . For  $q = 2$  and  $p = 5, 7$  we include the case  $\alpha = 3$  in order to show the coalescence of the first two negative extrema.

We close this section with a description of the methods of computation. The derivative of order  $r$  of the generalized Airy function may be written

$$(2.3) \quad Ai_{p,q}^{(r)}(\alpha, x) = \pi^{-1} \operatorname{Re} \int_0^\infty (iy)^r \exp\{iy^p/p - \alpha y^q/q + ixy\} dy,$$

$r = -1, 0, 1, \dots$ . In order to make the exponential decay as  $|y| \rightarrow \infty$ , we replace the path of integration by the ray in the complex plane

$$y = ze^{i\theta}, \quad 0 < z < \infty, \theta = \pi/(2(p + q)).$$

For  $r = -1$  we must add the contribution  $2\theta/\pi$  from the pole at the origin. For any given positive number  $\epsilon$  it is easy to find an  $R$  such that

$$\pi^{-1} \left| \int_R^\infty z^r \exp\{ie^{ip\theta} z^p/p - \alpha e^{iq\theta} z^q/q + ix e^{i\theta} z\} dz \right| < \epsilon.$$

We now use Romberg integration to compute the integral on the interval  $0 \leq z \leq R$ . In fact, the trapezoid rule for this integral is a finite complex Fourier series which is evaluated by the fast Fourier transform. The values of  $Ai_{p,q}$  and  $Aii_{p,q}$  were computed by this method. The values of  $F_q$  and  $Fi_q$  were computed in the same way but without the rotation of the path of integration. In order to find the local extrema of  $Aii_{p,q}$ , we used Newton's method, which requires evaluation of the integrals (2.3) for  $r = -1, 0, 1$ .

We checked our results as follows. First, it is easy to see that

$$Ai_{3,2}(\alpha, x) = \exp\{\alpha x/2\} Ai(x + \alpha^2/4),$$

where  $Ai$  is the usual Airy function. Second, the function  $Aii_{p,q}$  is a solution of the ordinary differential equation

$$(2.4) \quad (-1)^{(p-1)/2} u^{(p)} - \alpha(-1)^{q/2} u^{(q)} + xu' = 0,$$

which may be solved numerically once we have evaluated the integral (2.3) at a starting

value  $x$  for  $r = -1, 0, 1, \dots, \max(p, q) - 2$ . The difficulty with this method is that for  $q \geq 6$  and for  $p \geq 7$  (and also for  $p = 5$  if  $x < 0$ ) the differential equation (2.4) has a parasitic solution which dominates  $Aii_{p,q}$  as  $x$  increases and another parasitic solution which dominates  $Aii_{p,q}$  as  $x$  decreases. Hence, we must restart the computation fairly often. We found that it is better to start at a maximal value of  $|x|$  and go toward the origin, using a stiff equation solver [6].

We make a remark regarding our method of computing values of the integral (2.3). The method is adequate for the values of  $x$  we used, but for larger values of  $|x|$  there would be roundoff error from cancellation. We would then need to use a contour which better approximates the optimal path through the saddle points [4], [5].

**3. Applications to Difference Schemes.** We consider difference schemes for the equation

$$(3.1) \quad u_t + u_x = 0 \quad (-\infty < x < \infty, t > 0)$$

with  $u(x, 0) = \text{sgn}(x)/2$ . Our applications are based on the amplification function [9, p. 67] for the difference scheme. We assume that the difference scheme is stable. The amplification function has the form

$$G(\xi) = \exp\{\lambda(-i\xi + iA(\xi) - B(\xi))\},$$

where  $A$  and  $B$  are real, periodic functions such that

$$\begin{aligned} A(\xi) &= c\xi^p/p + O(|\xi|^{p+2}) \quad (\xi \rightarrow 0), \\ B(\xi) &= \epsilon\xi^q/q + O(\epsilon|\xi|^{q+2}) \quad (\xi \rightarrow 0). \end{aligned}$$

Here,  $p$  is odd,  $q$  is even,  $c \neq 0$ , and  $\lambda$  is the mesh ratio  $\lambda = \Delta t/h$ ,  $h = \Delta x$ . It may happen that  $B = 0$ , in which case the difference scheme is nondissipative. Otherwise, we have  $\epsilon > 0$ .

It was shown in [4] that if  $v_k^n$  denotes the solution of the difference scheme at the grid point  $x = kh$ ,  $t = n\Delta t$  with  $v_k^0 = \text{sgn}(kh)/2$ , then near the wave front we have the asymptotic estimate

$$(3.2) \quad \begin{aligned} V_k^n &\sim Aii_{p,q}(\alpha, y), \\ \alpha &= \epsilon|c|^{-q/p}(\lambda t/h)^{(p-q)/p}, \\ y &= \omega \text{sgn}(c)|c|^{-1/p}(t/h)^{(p-1)/p}. \end{aligned}$$

Here,  $\omega$  is the scaled distance from the wave front  $\omega = (x - t)/t$ . It was shown in [4] that the region of validity of (3.2) if  $q < p$  is

$$(3.3) \quad (\epsilon t/h)|c|^{-(pq+2)/(p^2-p)}|\omega|^{(q+2)/(p-1)} \rightarrow 0,$$

$$(3.4) \quad |\omega||c|^{-1/p}(t/h)^{(p-1)/(p+2)} \rightarrow 0.$$

If  $p < q$ , the region of validity of (3.2) is [4]

$$(3.5) \quad |\omega||c|^{-1/q}(t/h)^{(q-1)/(q+2)} \rightarrow 0$$

together with (3.3). Actually, the condition (3.3) was not needed in [4] because  $\epsilon$  was taken to be constant there; it is not hard to show by the methods of [4] that (3.3) is needed if  $\epsilon$  is allowed to vary. We remark that the numbers  $p, q, c$ , and  $\epsilon$  for a difference scheme may be obtained from the Maclaurin series for  $\log(G(\xi))$ .

If  $p$  and  $q$  are known,  $c$  and  $\epsilon$  may alternatively be obtained by a modification of the method used to find the local truncation error. This method is easier. One substitutes into the difference scheme a smooth solution  $u$  of the equation

$$(3.6) \quad u_t + u_x = (-1)^{(p-1)/2} cp^{-1} h^{p-1} \partial^p u / \partial x^p - (-1)^{q/2} \epsilon q^{-1} h^{q-1} \partial^q u / \partial x^q.$$

One then makes Taylor expansions, using Eq. (3.6) to eliminate derivatives with respect to  $t$ , and one chooses  $c$  and  $\epsilon$  so that the low-order terms cancel. We give an example in a moment. It is remarkable that this procedure is valid for a differential equation with a discontinuous solution, but it was shown in [4] and [5] that it is valid under conditions (3.3), (3.4) if  $q < p$  and under conditions (3.3), (3.5) if  $p < q$ . Chin [2] has found the error of the approximation in a special case.

We now give some examples.

*Example 1. Upstream differences.* For the upstream-difference method

$$(3.7) \quad v_k^{n+1} - v_k^n = \lambda(v_k^n - v_{k-1}^n)$$

we know that  $q = 2$  and  $p = 3$ . Upon substituting a smooth function  $u$  into Eq. (3.7) and using Taylor expansions, we obtain

$$(3.8) \quad \begin{aligned} u_t + (\lambda h/2)u_{tt} + (\lambda^2 h^2/6)u_{ttt} + O(h^3) \\ = -u_x + (h/2)u_{xx} - (h^2/6)u_{xxx} + O(h^3). \end{aligned}$$

In our case Eq. (3.6) becomes

$$u_t = -u_x + (\epsilon h/2)u_{xx} - (c h^2/3)u_{xxx},$$

so that

$$u_{tt} = u_{xx} - \epsilon h u_{xxx} + O(h^2), \quad u_{ttt} = -u_{xxx} + O(h).$$

Consequently, the terms of (3.8) cancel to within  $O(h^3)$  if and only if

$$\epsilon = 1 - \lambda, \quad c = (1 - \lambda)(1 - 2\lambda)/2.$$

Thus, if  $\lambda \neq 1/2$ , we have obtained  $\epsilon$  and  $c$  for the estimate (3.2) under conditions (3.3), (3.4). Note that  $p \geq 5$  if  $\lambda = 1/2$ .

*Example 2. The Lax-Wendroff method.* The Lax-Wendroff method [9, p. 331], when used for (3.1) has the amplification factor

$$G(\xi) = 1 - i\lambda \sin \xi - \lambda^2(1 - \cos \xi),$$

so that

$$\log G(\xi) = \lambda(i\xi + i(1 - \lambda^2)2^{-1}\xi^3/3 - \lambda(1 - \lambda^2)2^{-1}\xi^4/4 + O(\xi^5))$$

as  $\xi \rightarrow 0$ . Thus, we have (3.2) under conditions (3.3), (3.5) with  $p = 3$ ,  $q = 4$ .

$$c = (1 - \lambda^2)/2, \quad \epsilon = \lambda(1 - \lambda^2)/2.$$

*Example 3. Fromm's method.* Fromm's method [3] when applied to (3.1) has the amplification factor

$$G(\xi) = 1 - i(\lambda/2) \sin \xi(1 + e^{-i\xi}) - (\lambda/2)(\lambda + (\lambda - 2)e^{-i\xi})(1 - \cos \xi).$$

Consequently, we have

$$\log G(\xi) = \lambda(-i\xi + ic\xi^3/3 - \epsilon\xi^4/4 + O(\xi^5))$$

as  $\xi \rightarrow 0$  with

$$c = -(1 - \lambda)(1 - 2\lambda)/4, \quad \epsilon = (1 - \lambda)(1 - \lambda + \lambda^2)/2.$$

If  $0 < \lambda < 1/2$  or  $1/2 < \lambda < 1$ , we have (3.2) under conditions (3.3), (3.5) with  $p = 3$  and  $q = 4$ . If  $\lambda = 1/2$ , we have to determine  $p$  ( $p \geq 5$ ) and  $c$  by keeping higher order terms.

*Example 4. The leapfrog scheme.* The leapfrog scheme for (3.1) is

$$(3.9) \quad v_k^{n+1} - v_k^{n-1} = \lambda(v_{k+1}^n - v_{k-1}^n).$$

The solution to difference equation (3.9) has two waves, one with amplification factor

$$G(\xi) = -i\lambda \sin \xi + (1 - \lambda^2 \sin^2 \xi)^{1/2},$$

the other with amplification factor

$$G_2(\xi) = -i\lambda \sin \xi - (1 - \lambda^2 \sin^2 \xi)^{1/2}.$$

The wave generated by  $G_2$  has a front with speed  $-1$ , but it is easy to show that this wave has amplitude  $O(h^2)$  if the starting values  $v_k^0$  and  $v_k^1$  are correct to within  $O(h^2)$ . Consequently, we focus our attention on  $G_1$ . It is clear that for  $0 < \lambda < 1$  we have

$$|G_1(\xi)| = 1 \quad (-\infty < \xi < \infty).$$

Hence, there is no  $q$ -term, or equivalently,  $\alpha = 0$  in (3.2). The Maclaurin series for  $\log G_1(\xi)$  has the form

$$\log G_1(\xi) = \lambda(-i\xi + i(1 - \lambda^2)2^{-1}\xi^3/3 + O(\xi^5)).$$

Consequently, we have  $p = 3$  and

$$(3.10) \quad c = (1 - \lambda^2)/2.$$

Our asymptotic relation (3.2) is valid only if (3.4) holds, but because the difference scheme is not dissipative, the oscillations extend over a long distance. It is known [10] that the solution of (3.9) with initial data

$$v_k^0 = \text{sgn}(kh)/2, \quad v_k^1 = \text{sgn}(kh - \lambda h)/2$$

differs from the exact solution to (3.1) by  $O((h/t)^{1/2})$  over every interval

$$|x| < \theta t \quad (0 < \theta < 1).$$

In order to reduce this error, one usually adds an artificial viscosity. If this artificial viscosity is linear, the model equation (3.6) takes the form

$$u_t + u_x = -(ch^2/3)u_{xxx} - \epsilon(-1)^{q/2}q^{-1}h^{q-1}\partial^q u/\partial x^q.$$

One ordinarily chooses  $q = 2$  or  $q = 4$ . If  $q = 2$  and  $\epsilon = \delta h^{1/3}$  for a fixed constant  $\delta$ , then (3.2) takes the form

$$\begin{aligned} v_k^n &\sim Aii_{3,2}(\alpha, y), \\ \alpha &= \delta c^{-2/3}(\lambda t)^{1/3}, \\ y &= (x - t)(ct)^{-1/3}h^{2/3} \end{aligned}$$

with  $c$  as in (3.10). Note that this choice of  $\epsilon$  makes  $\alpha$  independent of  $h$ . The artificial viscosity is a discretization of  $(\delta h^{4/3}/2)u_{xx}$ . Similarly, if we want  $q = 4$  and

$$v_k^n \sim Aii_{3,4}(\alpha, y)$$

with  $\alpha$  independent of  $h$ , we take  $\epsilon = \delta h^{1/3}$ . The artificial viscosity is then a discretization of  $-(\delta h^{8/3}/4)u_{xxxx}$ .

*Example 5. Fourth-order schemes.* For a fourth-order, nondissipative scheme Eq. (3.2) takes the form

$$v_k^n \sim Aii_{5,q}(0, y)$$

under condition (3.4) with  $p = 5$ . Here,

$$y = (x - t)(|c|t)^{-1/5} \operatorname{sgn}(c)h^{-4/5}.$$

If we want positive  $\alpha$  independent of  $h$ , we add to the difference scheme an artificial viscosity which is a difference approximation to one of the following

$$\begin{aligned} \delta h^{1.6} \partial^2 u / \partial x^2 & \quad (q = 2), \\ -\delta h^{3.2} \partial^4 u / \partial x^4 & \quad (q = 4), \\ \delta h^{4.8} \partial^6 u / \partial x^6 & \quad (q = 6), \dots \end{aligned}$$

Other examples of model equations (3.6) are given in [11].

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R. C. Y. Chin and G. W. Hedstrom, A Dispersion Analysis for Difference Schemes: Tables of Generalized Airy Functions, p. 1163.

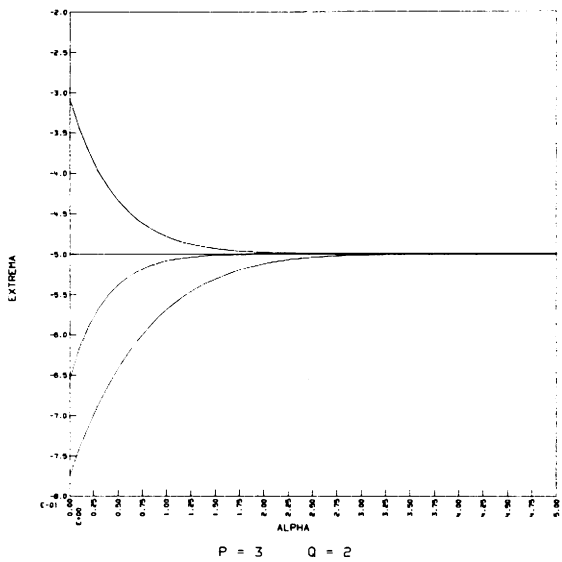
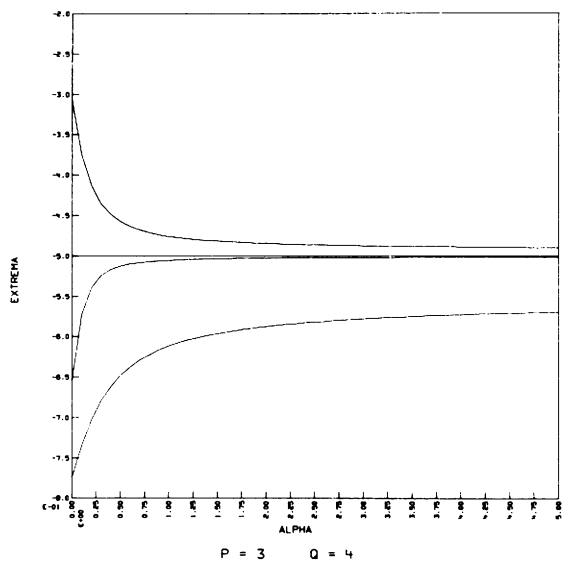


Figure 1



P = 3    Q = 4

Figure 2

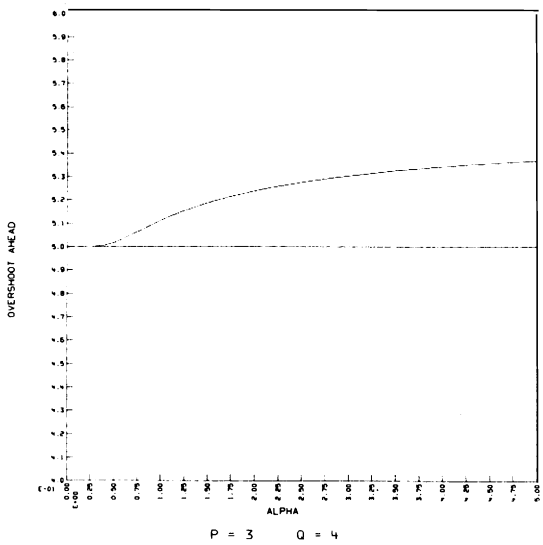
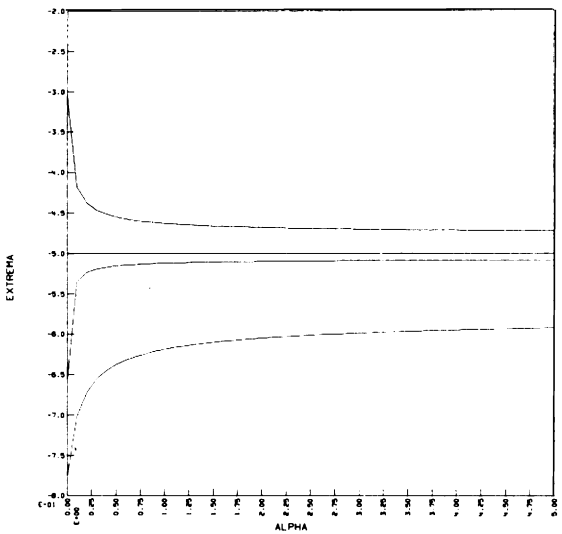
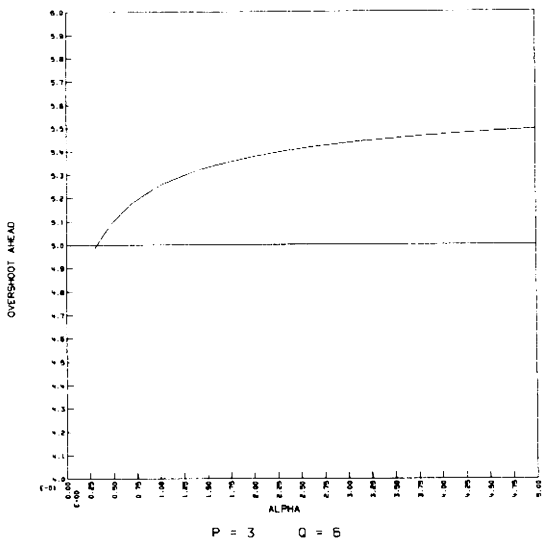


Figure 3



P = 3    Q = 6

Figure 4



P = 3    Q = 6

Figure 5

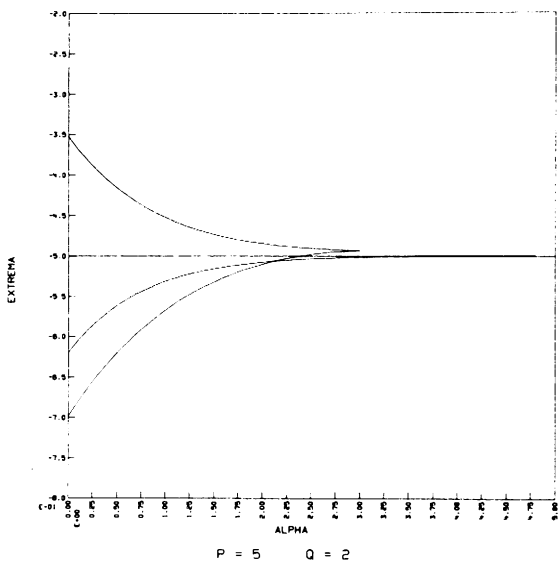


Figure 6

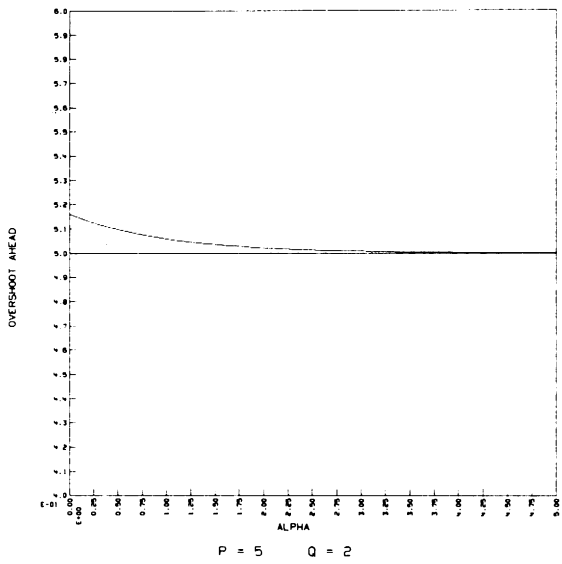


Figure 7

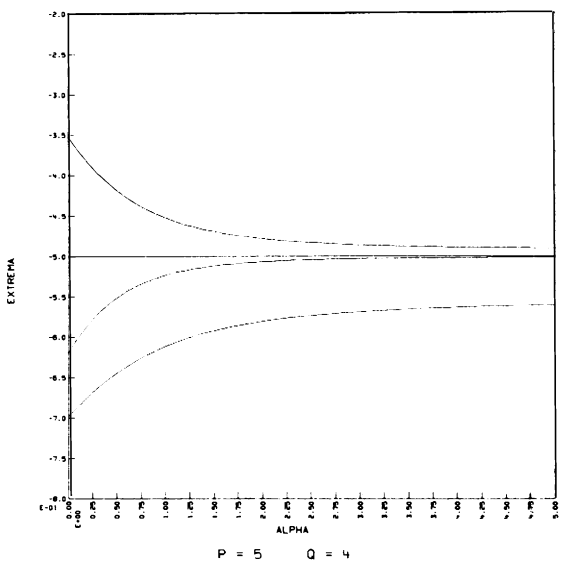
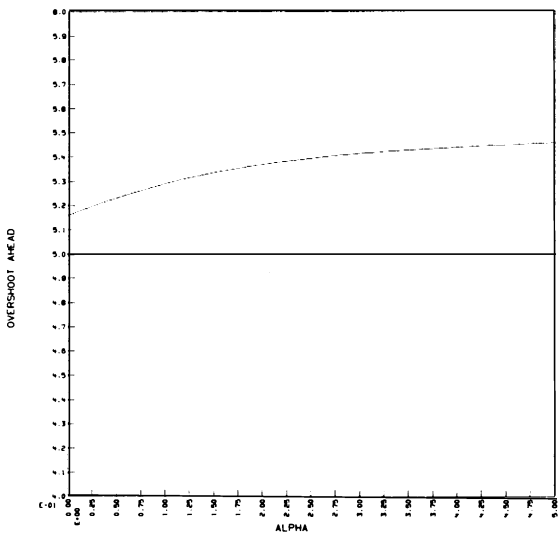


Figure 8





P = 5    Q = 4

Figure 9

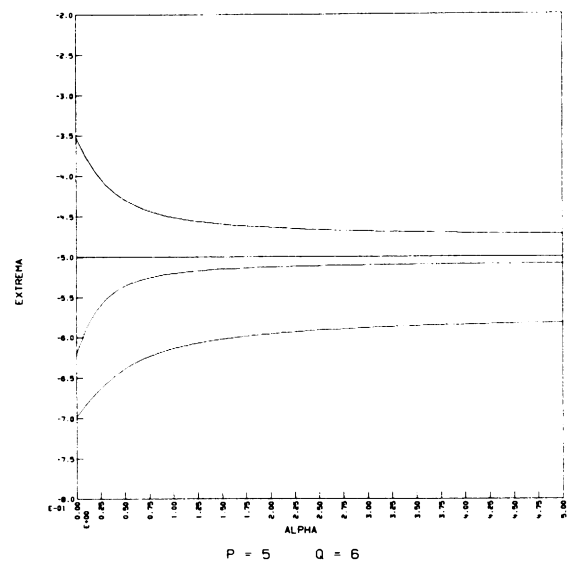
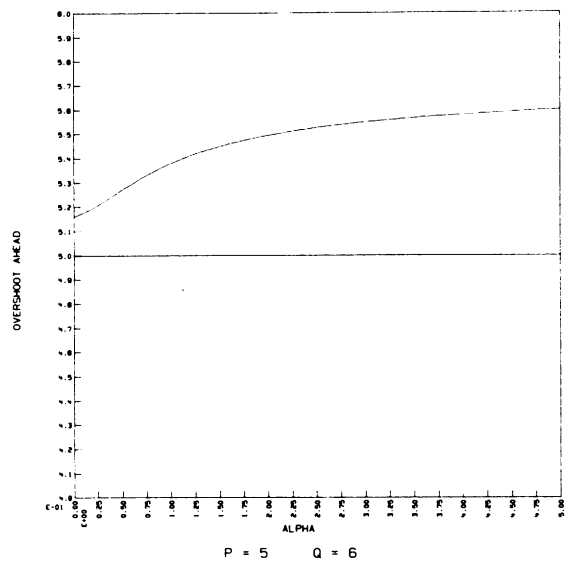
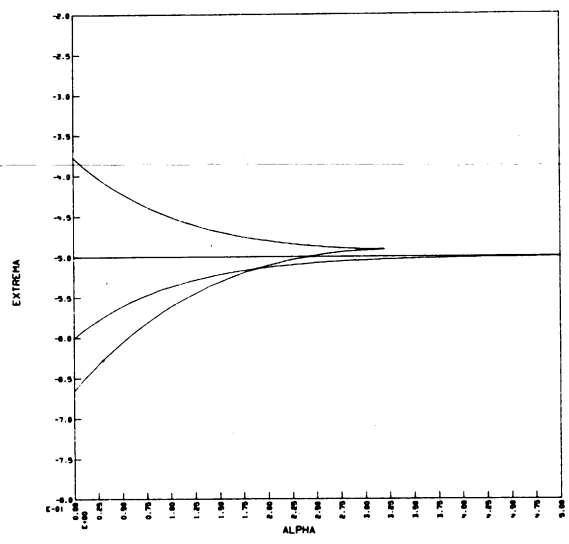


Figure 10



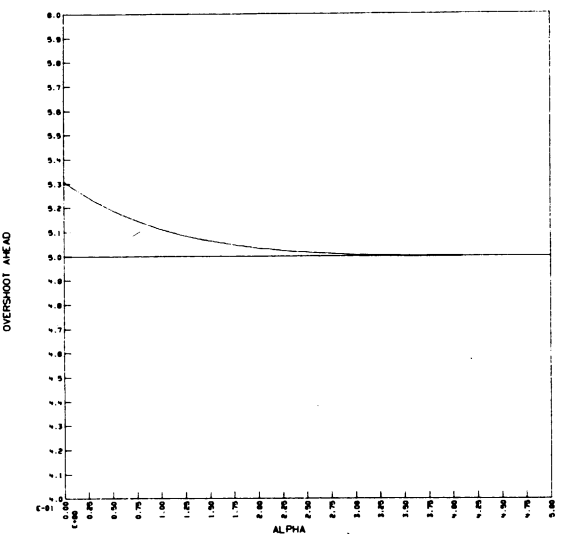
P = 5    Q = 6

Figure 11



P = 7    Q = 2

Figure 12



P = 7    Q = 2

Figure 13

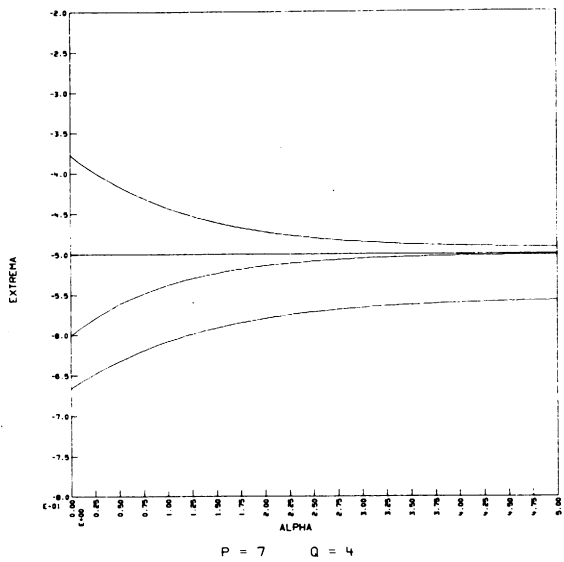
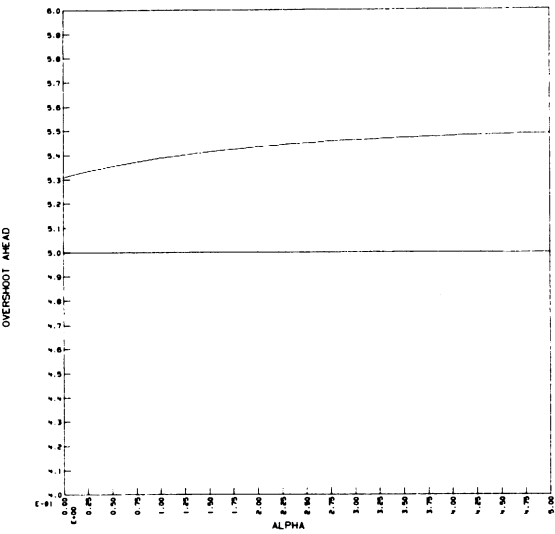
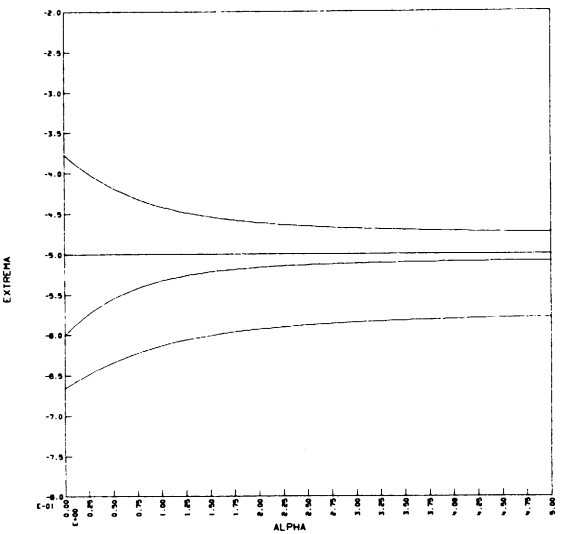


Figure 14



P = 7    Q = 4

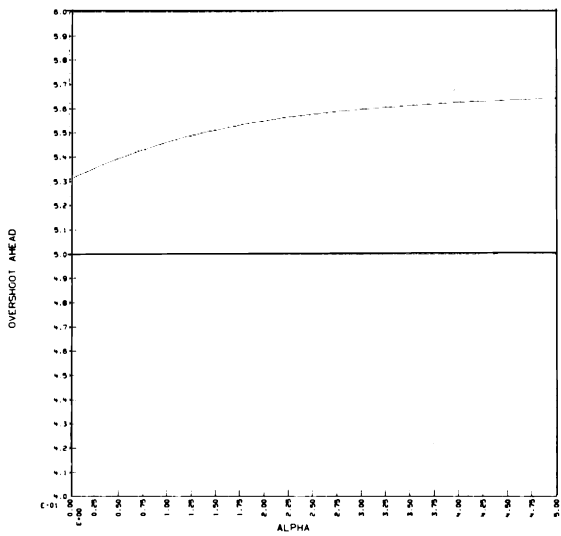
Figure 15



P = 7    Q = 6

Figure 16





P = 7    Q = 6

Figure 17

Table 1

PAGE 1

EXTREMUM FOR THE INTEGRAL OF THE GENERALIZED AIRY FUNCTION P = 3 Q = 2

ALPHA	B	A11(1)	A	A11(1)	K	A11(1)	ALPHA
0	-2	3.88197e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.89507e-30
0.10	-2	3.88607e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.817218e-0
0.20	-2	3.89197e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.589512e-0
0.30	-2	3.90007e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.286796e-0
0.40	-2	3.91197e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.563136e-0
0.50	-2	3.92807e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.217681e-0
0.60	-2	3.94807e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.543873e-0
0.70	-2	3.97207e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
0.80	-2	3.99907e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.515671e-0
0.90	-2	4.02907e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
1.00	-2	4.06207e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
1.10	-2	4.09807e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
1.20	-2	4.13707e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
1.30	-2	4.17907e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
1.40	-2	4.22407e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
1.50	-2	4.27207e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
1.60	-2	4.32307e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
1.70	-2	4.37707e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
1.80	-2	4.43407e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
1.90	-2	4.49407e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
2.00	-2	4.55707e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
2.10	-2	4.62307e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
2.20	-2	4.69207e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
2.30	-2	4.76407e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
2.40	-2	4.83907e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
2.50	-2	4.91707e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
2.60	-2	4.99807e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
2.70	-2	5.08207e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
2.80	-2	5.16907e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
2.90	-2	5.25907e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
3.00	-2	5.35207e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
3.10	-2	5.44807e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
3.20	-2	5.54707e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
3.30	-2	5.64907e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
3.40	-2	5.75407e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
3.50	-2	5.86207e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
3.60	-2	5.97307e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
3.70	-2	6.08707e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
3.80	-2	6.20407e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
3.90	-2	6.32407e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
4.00	-2	6.44707e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
4.10	-2	6.57307e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
4.20	-2	6.70207e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
4.30	-2	6.83407e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
4.40	-2	6.96907e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
4.50	-2	7.10707e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
4.60	-2	7.24807e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
4.70	-2	7.39207e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
4.80	-2	7.53907e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0
4.90	-2	7.68907e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.211801e-0
5.00	-2	7.84207e-1	0.778326e-1	-0.0778326e-1	-1	5.25959e-1	-0.500000e-0

Table 2

EXTREMA FOR THE INTEGRAL OF THE GENERALIZED AIRY FUNCTION										
Alpha	A1111			A1112			A1113			Alpha
	x	f	f'	x	f	f'	x	f	f'	
0.00	-2.33819761	-0.77620968	-0.82704004	-3.32682288	-0.52039081	-0.69503436				0.00
0.10	-2.33328218	-0.77620968	-0.82704004	3.774327	-0.504183	-0.57323867				0.10
0.20	-2.33017650	-0.77620968	-0.82704004	4.135649	-0.6631907	-0.73826361	0.06691	0	4.000328	0.20
0.30	-2.32862552	-0.77620968	-0.82704004	4.477971	-0.999573	-0.999573	0.263823	0	4.000328	0.30
0.40	-2.32822458	-0.81188146	-0.82704004	4.800000	-1.4847129	-1.4847129	0.693180	3.7812122	4.000328	0.40
0.50	-2.32898811	-0.84753876	-0.82704004	5.1101015	-2.1200122	-2.1200122	1.2917000	6.0939170	4.000328	0.50
0.60	-2.33009993	-0.88319406	-0.82704004	5.4048179	-2.999573	-2.999573	2.099573	9.999573	4.000328	0.60
0.70	-2.33270988	-0.91884936	-0.82704004	5.690000	-4.135649	-4.135649	3.135649	14.135649	4.000328	0.70
0.80	-2.33600000	-0.95450466	-0.82704004	6.000000	-5.6631907	-5.6631907	4.6631907	21.6631907	4.000328	0.80
0.90	-2.34000000	-0.99016000	-0.82704004	6.333333	-7.500000	-7.500000	6.000000	32.500000	4.000328	0.90
1.00	-2.34500000	-1.02581530	-0.82704004	6.700000	-9.666666	-9.666666	7.666666	48.666666	4.000328	1.00
1.10	-2.35100000	-1.06147060	-0.82704004	7.100000	-12.133333	-12.133333	10.333333	70.333333	4.000328	1.10
1.20	-2.35800000	-1.09712590	-0.82704004	7.533333	-14.916666	-14.916666	13.333333	98.333333	4.000328	1.20
1.30	-2.36600000	-1.13278120	-0.82704004	8.000000	-18.000000	-18.000000	16.666666	133.333333	4.000328	1.30
1.40	-2.37500000	-1.16843650	-0.82704004	8.500000	-21.428571	-21.428571	20.357142	166.428571	4.000328	1.40
1.50	-2.38500000	-1.20409180	-0.82704004	9.033333	-25.200000	-25.200000	24.333333	207.666666	4.000328	1.50
1.60	-2.39600000	-1.24074710	-0.82704004	9.600000	-29.333333	-29.333333	28.666666	257.000000	4.000328	1.60
1.70	-2.40800000	-1.27840240	-0.82704004	10.200000	-33.833333	-33.833333	33.333333	314.666666	4.000328	1.70
1.80	-2.42100000	-1.31705770	-0.82704004	10.833333	-38.700000	-38.700000	38.333333	380.666666	4.000328	1.80
1.90	-2.43500000	-1.35671300	-0.82704004	11.500000	-43.966666	-43.966666	43.333333	455.333333	4.000328	1.90
2.00	-2.45000000	-1.39736830	-0.82704004	12.200000	-49.633333	-49.633333	49.000000	539.000000	4.000328	2.00
2.10	-2.46600000	-1.43902360	-0.82704004	12.933333	-55.700000	-55.700000	54.333333	631.666666	4.000328	2.10
2.20	-2.48300000	-1.48167890	-0.82704004	13.700000	-62.166666	-62.166666	60.000000	733.333333	4.000328	2.20
2.30	-2.50100000	-1.52533420	-0.82704004	14.500000	-69.033333	-69.033333	67.000000	855.000000	4.000328	2.30
2.40	-2.52000000	-1.56998950	-0.82704004	15.333333	-76.300000	-76.300000	74.333333	997.666666	4.000328	2.40
2.50	-2.54000000	-1.61564480	-0.82704004	16.200000	-84.066666	-84.066666	82.000000	1162.333333	4.000328	2.50
2.60	-2.56100000	-1.66230010	-0.82704004	17.100000	-92.333333	-92.333333	90.333333	1350.000000	4.000328	2.60
2.70	-2.58300000	-1.70995540	-0.82704004	18.033333	-101.100000	-101.100000	99.000000	1561.666666	4.000328	2.70
2.80	-2.60600000	-1.75861070	-0.82704004	19.000000	-110.366666	-110.366666	108.000000	1797.333333	4.000328	2.80
2.90	-2.63000000	-1.80826600	-0.82704004	19.933333	-120.133333	-120.133333	117.333333	2058.000000	4.000328	2.90
3.00	-2.65500000	-1.85892130	-0.82704004	20.900000	-130.400000	-130.400000	127.000000	2343.666666	4.000328	3.00
3.10	-2.68100000	-1.91057660	-0.82704004	21.900000	-141.166666	-141.166666	137.333333	2654.333333	4.000328	3.10
3.20	-2.70800000	-1.96323190	-0.82704004	22.933333	-152.433333	-152.433333	148.000000	3000.000000	4.000328	3.20
3.30	-2.73600000	-2.01688720	-0.82704004	24.000000	-164.200000	-164.200000	159.000000	3390.666666	4.000328	3.30
3.40	-2.76500000	-2.07154250	-0.82704004	25.100000	-176.466666	-176.466666	170.333333	3826.333333	4.000328	3.40
3.50	-2.79500000	-2.12719780	-0.82704004	26.233333	-189.233333	-189.233333	182.000000	4307.000000	4.000328	3.50
3.60	-2.82600000	-2.18385310	-0.82704004	27.400000	-202.500000	-202.500000	194.333333	4832.666666	4.000328	3.60
3.70	-2.85800000	-2.24150840	-0.82704004	28.600000	-216.266666	-216.266666	207.000000	5403.333333	4.000328	3.70
3.80	-2.89100000	-2.30016370	-0.82704004	29.833333	-230.533333	-230.533333	220.333333	6019.000000	4.000328	3.80
3.90	-2.92500000	-2.35981900	-0.82704004	31.100000	-245.300000	-245.300000	234.333333	6680.666666	4.000328	3.90
4.00	-2.96000000	-2.42047430	-0.82704004	32.400000	-260.566666	-260.566666	248.666666	7398.333333	4.000328	4.00

Table 3

EXTREMUM FOR THE INTEGRAL OF THE GENERALIZED AIRY FUNCTION

Alpha	h	A1111	k	A1114	h	A1114	h	A1114	k	A1111	A.Pha
0.00	2.3810741	0.7792506	0.0879094	0.1024798	-0.5205981	0.0201336	0.0000000	0.0000000	0.0000000	0.0000000	2.10
0.10	2.2870031	0.7207025	0.1193803	-0.171762	-0.5321148	0.0131175	0.0000000	0.0000000	0.0000000	0.0000000	2.10
0.20	2.2041876	0.6717482	0.1481095	-0.2801809	-0.5311277	0.0095471	0.0000000	0.0000000	0.0000000	0.0000000	2.10
0.30	2.1306427	0.6309179	0.1717051	-0.4073907	-0.5289518	0.0075280	0.0000000	0.0000000	0.0000000	0.0000000	2.10
0.40	2.0681273	0.6011095	0.1929272	-0.5181956	-0.5269208	0.0064700	0.0000000	0.0000000	0.0000000	0.0000000	2.10
0.50	2.0120509	0.5787031	0.2117178	-0.6110941	-0.5250089	0.0058980	0.0000000	0.0000000	0.0000000	0.0000000	2.10
0.60	2.0617130	0.5623327	0.2287010	-0.6761295	-0.5238240	0.0056170	0.0000000	0.0000000	0.0000000	0.0000000	2.10
0.70	2.0204880	0.5477081	0.2438122	-0.7269709	-0.5231646	0.0054670	0.0000000	0.0000000	0.0000000	0.0000000	2.10
0.80	2.0297099	0.5341137	0.2580988	-0.6991901	-0.5230000	0.0054222	0.0000000	0.0000000	0.0000000	0.0000000	2.10
0.90	2.0681017	0.5211309	0.2709873	-0.6211168	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
1.00	2.0999089	0.5100073	0.2823116	-0.6101713	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
1.10	2.1170410	0.5181700	0.2920320	-0.6199464	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
1.20	2.1154688	0.5147988	0.2993357	-0.6466337	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
1.30	2.0974044	0.5120488	0.3053886	-0.6532648	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
1.40	2.1164879	0.51198182	0.3093268	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
1.50	2.0974044	0.5120488	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
1.60	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
1.70	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
1.80	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
1.90	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
2.00	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
2.10	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
2.20	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
2.30	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
2.40	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
2.50	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
2.60	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
2.70	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
2.80	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
2.90	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
3.00	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
3.10	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
3.20	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
3.30	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
3.40	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
3.50	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
3.60	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
3.70	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
3.80	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
3.90	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10
4.00	2.0662078	0.5097524	0.3093357	-0.6690240	-0.5229849	0.0054169	0.0000000	0.0000000	0.0000000	0.0000000	2.10

Table 4

EXTREMUM FOR THE INTEGRAL OF THE GENERALIZED AIRY FUNCTION:  $P = 5 \ 0 \ 2$

ALPHA	X	ALLI(X)	Y	ALLI(X)	ALPHA
0.00	2.4326205	0.6882381	7.0551670	0.111111	0
0.10	2.4495787	0.6882395	7.0556670	0.2474783	0.1108811
0.20	2.5059756	0.6832819	7.1219316	0.3852078	0.1106950
0.30	2.5711462	0.6698738	7.2027774	0.5230796	0.1105209
0.40	2.6473381	0.6385289	7.2977720	0.6604437	0.1103588
0.50	2.7311886	0.6013381	7.4062748	0.7967919	0.1102088
0.60	2.8207445	0.5679810	7.5286796	0.9317291	0.1100700
0.70	2.9133366	0.5380789	7.6654906	1.0657749	0.1099425
0.80	2.9792670	0.5122370	7.8173788	1.2084798	0.1098254
0.90	2.9993966	0.4898807	7.9841708	1.3593997	0.1097178
1.00	2.9999911	0.4709078	8.1658113	1.5181174	0.1096189
1.10	2.9999995	0.4546694	8.3624409	1.6843185	0.1095280
1.20	2.9999995	0.4406287	8.5741488	1.8575179	0.1094444
1.30	2.9999995	0.4283304	8.8009480	2.0373188	0.1093684
1.40	2.9999995	0.4173304	9.0423480	2.2233188	0.1093000
1.50	2.9999995	0.4073304	9.2983480	2.4151188	0.1092392
1.60	2.9999995	0.4000000	9.5689480	2.6123188	0.1091860
1.70	2.9999995	0.3940000	9.8541480	2.8155188	0.1091404
1.80	2.9999995	0.3890000	10.1539480	3.0243188	0.1091014
1.90	2.9999995	0.3850000	10.4683480	3.2383188	0.1090688
2.00	2.9999995	0.3820000	10.7973480	3.4571188	0.1090416
2.10	2.9999995	0.3790000	11.1409480	3.6805188	0.1090188
2.20	2.9999995	0.3760000	11.4993480	3.9083188	0.1090004
2.30	2.9999995	0.3730000	11.8725480	4.1403188	0.1089854
2.40	2.9999995	0.3700000	12.2605480	4.3763188	0.1089736
2.50	2.9999995	0.3670000	12.6633480	4.6161188	0.1089648
2.60	2.9999995	0.3640000	13.0809480	4.8595188	0.1089588
2.70	2.9999995	0.3610000	13.5133480	5.1063188	0.1089552
2.80	2.9999995	0.3580000	13.9605480	5.3563188	0.1089536
2.90	2.9999995	0.3550000	14.4225480	5.6093188	0.1089536
3.00	2.9999995	0.3520000	14.8993480	5.8651188	0.1089552
3.10	2.9999995	0.3490000	15.3909480	6.1235188	0.1089584
3.20	2.9999995	0.3460000	15.8973480	6.3843188	0.1089632
3.30	2.9999995	0.3430000	16.4185480	6.6473188	0.1089696
3.40	2.9999995	0.3400000	16.9545480	6.9123188	0.1089776
3.50	2.9999995	0.3370000	17.5053480	7.1791188	0.1089872
3.60	2.9999995	0.3340000	18.0709480	7.4475188	0.1089984
3.70	2.9999995	0.3310000	18.6513480	7.7173188	0.1090112
3.80	2.9999995	0.3280000	19.2465480	7.9883188	0.1090256
3.90	2.9999995	0.3250000	19.8565480	8.2603188	0.1090416
4.00	2.9999995	0.3220000	20.4813480	8.5331188	0.1090592
4.10	2.9999995	0.3190000	21.1209480	8.8065188	0.1090784
4.20	2.9999995	0.3160000	21.7753480	9.0803188	0.1090992
4.30	2.9999995	0.3130000	22.4445480	9.3543188	0.1091216
4.40	2.9999995	0.3100000	23.1285480	9.6283188	0.1091456
4.50	2.9999995	0.3070000	23.8273480	9.9023188	0.1091712
4.60	2.9999995	0.3040000	24.5409480	10.1763188	0.1091984
4.70	2.9999995	0.3010000	25.2693480	10.4503188	0.1092272
4.80	2.9999995	0.2980000	26.0125480	10.7243188	0.1092576
4.90	2.9999995	0.2950000	26.7705480	11.0000000	0.1092896
5.00	2.9999995	0.2920000	27.5433480	11.2771188	0.1093232

Table 5

ALPHA											
$x$	$y$	$z$	$w$	$v$	$u$	$t$	$s$	$r$	$q$	$p$	$o$
0	2.47326205	-0.88823873	-0.70001679	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0
0.10	-2.4701879	-0.69561785	-0.70270997	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0
0.20	-2.46691373	-0.51399251	-0.71074069	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0
0.30	-2.46347536	-0.34333180	-0.72000000	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0
0.40	-2.45993290	-0.18545862	-0.73000000	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0
0.50	-2.45633098	-0.04773944	-0.74000000	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0
0.60	-2.45271761	0.07877948	-0.75000000	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0
0.70	-2.44914295	0.20773944	-0.76000000	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0
0.80	-2.44564709	0.34333180	-0.77000000	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0
0.90	-2.44226095	0.48444444	-0.78000000	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0
1.00	-2.43891461	0.63000000	-0.79000000	-0.35426783	-0.72601722	-0.61082031	2.79429427	0.51508413	0	0	0

Table 6

ALPHA	X	Y	ALPHA	X	Y	ALPHA	X	Y	ALPHA	X	Y
0.0	0.000000	0.000000	0.0	0.000000	0.000000	0.0	0.000000	0.000000	0.0	0.000000	0.000000
0.10	0.000000	0.000000	0.10	0.000000	0.000000	0.10	0.000000	0.000000	0.10	0.000000	0.000000
0.20	0.000000	0.000000	0.20	0.000000	0.000000	0.20	0.000000	0.000000	0.20	0.000000	0.000000
0.30	0.000000	0.000000	0.30	0.000000	0.000000	0.30	0.000000	0.000000	0.30	0.000000	0.000000
0.40	0.000000	0.000000	0.40	0.000000	0.000000	0.40	0.000000	0.000000	0.40	0.000000	0.000000
0.50	0.000000	0.000000	0.50	0.000000	0.000000	0.50	0.000000	0.000000	0.50	0.000000	0.000000
0.60	0.000000	0.000000	0.60	0.000000	0.000000	0.60	0.000000	0.000000	0.60	0.000000	0.000000
0.70	0.000000	0.000000	0.70	0.000000	0.000000	0.70	0.000000	0.000000	0.70	0.000000	0.000000
0.80	0.000000	0.000000	0.80	0.000000	0.000000	0.80	0.000000	0.000000	0.80	0.000000	0.000000
0.90	0.000000	0.000000	0.90	0.000000	0.000000	0.90	0.000000	0.000000	0.90	0.000000	0.000000
1.00	0.000000	0.000000	1.00	0.000000	0.000000	1.00	0.000000	0.000000	1.00	0.000000	0.000000

Table 7

EXTREMAL VALUES FOR THE INTEGRAL OF THE GENERALIZED AIRY FUNCTION

$\alpha$	$x$	$y$	$x$	$y$	$x$	$y$	$x$	$y$	$x$	$y$	
7	2	0.9110662	0.8681013	4	0.7053962	0.3776396	0.27766399	0.6058911	2	0.9510296	0.5106679
7	10	0.7162961	0.6574810	4	0.8086972	0.3881795	0.3122171	2	1.016049	0.5200565	0
7	25	0.5071390	0.4396262	4	0.8130881	0.3978782	0.2875583	2	1.0896287	0.5276783	20
7	50	0.3709660	0.3276379	4	0.8095063	0.4069961	0.2599960	2	1.1743959	0.5351376	30
7	75	0.2823115	0.246277	5	0.8159403	0.4149893	0.2399931	2	1.2688359	0.5429390	40
7	90	0.2200398	0.1877681	5	0.8172794	0.4225297	0.2299130	2	1.3739926	0.5510936	50
7	95	0.2013365	0.1698619	5	0.8172794	0.4225297	0.2299130	2	1.4898753	0.5595223	70
7	97	0.1788953	0.1566017	5	0.8166798	0.4213499	0.2299130	2	1.6166795	0.5681909	80
7	98	0.1689583	0.1477919	5	0.8151791	0.4191133	0.2299130	2	1.7549326	0.5771499	90
7	99	0.16178139	0.1409937	5	0.8127153	0.4158195	0.2299130	2	1.9041110	0.5863510	100
7	99	0.1567995	0.1347908	5	0.8093761	0.4114766	0.2299130	2	2.0649391	0.5957500	110
7	99	0.1527995	0.1290902	5	0.8051992	0.4061766	0.2299130	2	2.2379299	0.6053015	120
7	99	0.1497995	0.1237908	5	0.8001992	0.4001766	0.2299130	2	2.4234995	0.6150515	130
7	99	0.1477995	0.1187908	5	0.7943992	0.3934992	0.2299130	2	2.6211995	0.6250515	140
7	99	0.1467995	0.1139908	5	0.7877992	0.3861992	0.2299130	2	2.8314995	0.6352515	150
7	99	0.1467995	0.1092908	5	0.7803992	0.3783992	0.2299130	2	3.0541995	0.6456015	160
7	99	0.1467995	0.1046908	5	0.7721992	0.3701992	0.2299130	2	3.2891995	0.6560515	170
7	99	0.1467995	0.1001908	5	0.7631992	0.3615992	0.2299130	2	3.5361995	0.6666015	180
7	99	0.1467995	0.0956908	5	0.7533992	0.3525992	0.2299130	2	3.7941995	0.6772515	190
7	99	0.1467995	0.0911908	5	0.7427992	0.3431992	0.2299130	2	4.0631995	0.6879515	200
7	99	0.1467995	0.0866908	5	0.7313992	0.3333992	0.2299130	2	4.3431995	0.6986515	210
7	99	0.1467995	0.0821908	5	0.7191992	0.3231992	0.2299130	2	4.6341995	0.7093515	220
7	99	0.1467995	0.0776908	5	0.7061992	0.3125992	0.2299130	2	4.9361995	0.7200515	230
7	99	0.1467995	0.0731908	5	0.6923992	0.3015992	0.2299130	2	5.2491995	0.7307515	240
7	99	0.1467995	0.0686908	5	0.6777992	0.2901992	0.2299130	2	5.5731995	0.7414515	250
7	99	0.1467995	0.0641908	5	0.6623992	0.2783992	0.2299130	2	5.9081995	0.7521515	260
7	99	0.1467995	0.0596908	5	0.6461992	0.2661992	0.2299130	2	6.2541995	0.7628515	270
7	99	0.1467995	0.0551908	5	0.6291992	0.2535992	0.2299130	2	6.6111995	0.7735515	280
7	99	0.1467995	0.0506908	5	0.6113992	0.2405992	0.2299130	2	6.9791995	0.7842515	290
7	99	0.1467995	0.0461908	5	0.5927992	0.2271992	0.2299130	2	7.3581995	0.7949515	300
7	99	0.1467995	0.0416908	5	0.5733992	0.2133992	0.2299130	2	7.7491995	0.8056515	310
7	99	0.1467995	0.0371908	5	0.5531992	0.1991992	0.2299130	2	8.1521995	0.8163515	320
7	99	0.1467995	0.0326908	5	0.5321992	0.1845992	0.2299130	2	8.5671995	0.8270515	330
7	99	0.1467995	0.0281908	5	0.5103992	0.1695992	0.2299130	2	8.9941995	0.8377515	340
7	99	0.1467995	0.0236908	5	0.4877992	0.1541992	0.2299130	2	9.4331995	0.8484515	350
7	99	0.1467995	0.0191908	5	0.4643992	0.1383992	0.2299130	2	9.8841995	0.8591515	360
7	99	0.1467995	0.0146908	5	0.4401992	0.1221992	0.2299130	2	10.3471995	0.8698515	370
7	99	0.1467995	0.0101908	5	0.4151992	0.1055992	0.2299130	2	10.8221995	0.8805515	380
7	99	0.1467995	0.0056908	5	0.3893992	0.0885992	0.2299130	2	11.3091995	0.8912515	390
7	99	0.1467995	0.0011908	5	0.3627992	0.0711992	0.2299130	2	11.8081995	0.9019515	400
7	99	0.1467995	0.0000000	5	0.3353992	0.0533992	0.2299130	2	12.3191995	0.9126515	410
7	99	0.1467995	0.0000000	5	0.3071992	0.0351992	0.2299130	2	12.8421995	0.9233515	420
7	99	0.1467995	0.0000000	5	0.2781992	0.0165992	0.2299130	2	13.3771995	0.9340515	430
7	99	0.1467995	0.0000000	5	0.2483992	0.0000000	0.2299130	2	13.9241995	0.9447515	440
7	99	0.1467995	0.0000000	5	0.2177992	0.0000000	0.2299130	2	14.4831995	0.9554515	450
7	99	0.1467995	0.0000000	5	0.1863992	0.0000000	0.2299130	2	15.0541995	0.9661515	460
7	99	0.1467995	0.0000000	5	0.1541992	0.0000000	0.2299130	2	15.6371995	0.9768515	470
7	99	0.1467995	0.0000000	5	0.1211992	0.0000000	0.2299130	2	16.2321995	0.9875515	480
7	99	0.1467995	0.0000000	5	0.0883992	0.0000000	0.2299130	2	16.8391995	0.9982515	490
7	99	0.1467995	0.0000000	5	0.0557992	0.0000000	0.2299130	2	17.4581995	1.0089515	500



Table 8

EXTrema FOR THE INTEGRAL OF THE GENERALIZED AIRY FUNCTION.  $\mu = 7.0 - \epsilon$

ALPHA	A1(1)	A1(1)	A1(1)	A1(1)	A1(1)	A1(1)	ALPHA
0.00	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000
0.02	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0200000
0.04	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0400000
0.06	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0600000
0.08	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0800000
0.10	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.1000000
0.12	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.1200000
0.14	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.1400000
0.16	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.1600000
0.18	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.1800000
0.20	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.2000000
0.22	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.2200000
0.24	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.2400000
0.26	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.2600000
0.28	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.2800000
0.30	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.3000000
0.32	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.3200000
0.34	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.3400000
0.36	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.3600000
0.38	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.3800000
0.40	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.4000000
0.42	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.4200000
0.44	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.4400000
0.46	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.4600000
0.48	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.4800000
0.50	0.0000000	0.0000000	-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.5000000



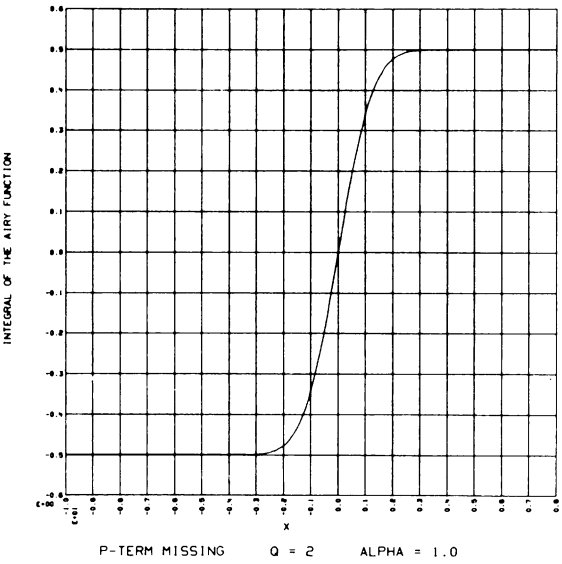


Figure 18

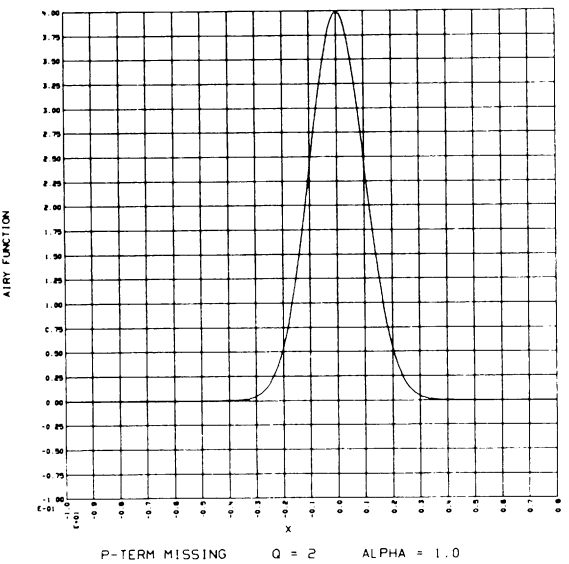


Figure 19

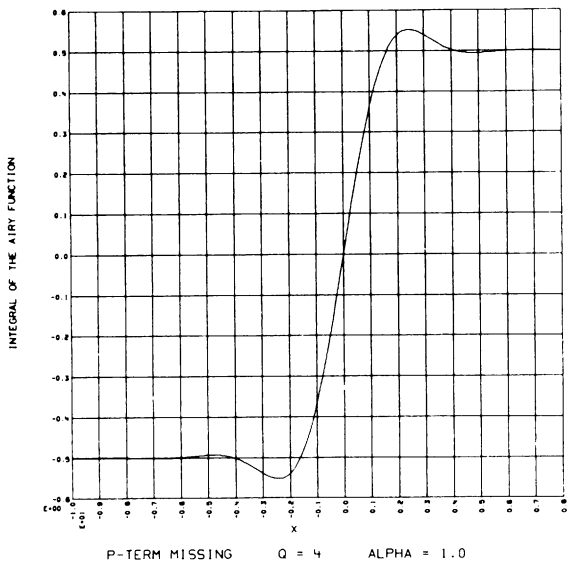


Figure 20

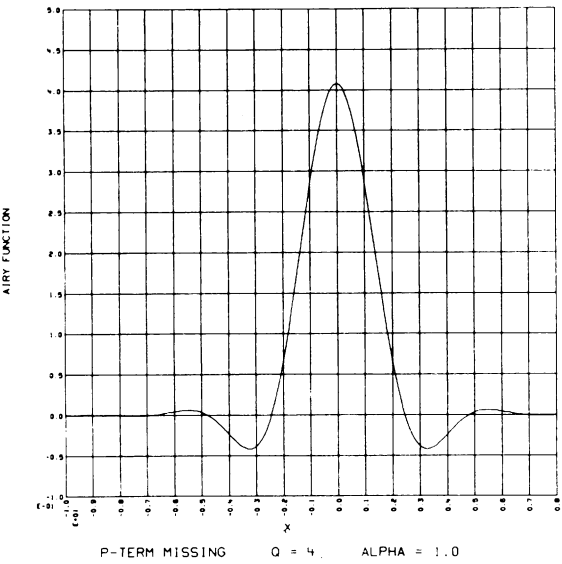


Figure 21

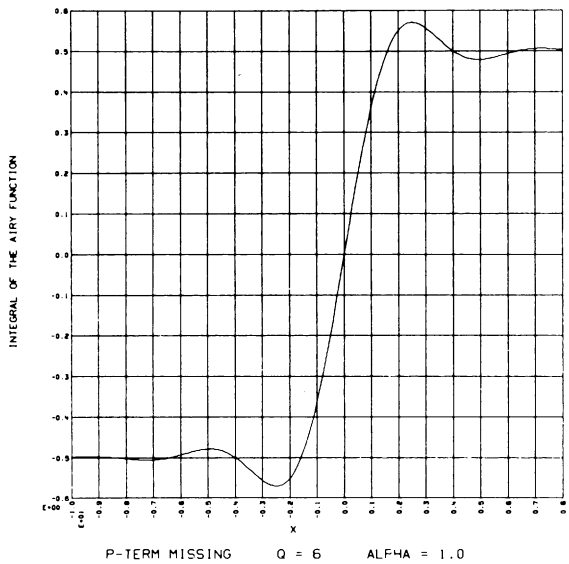


Figure 22

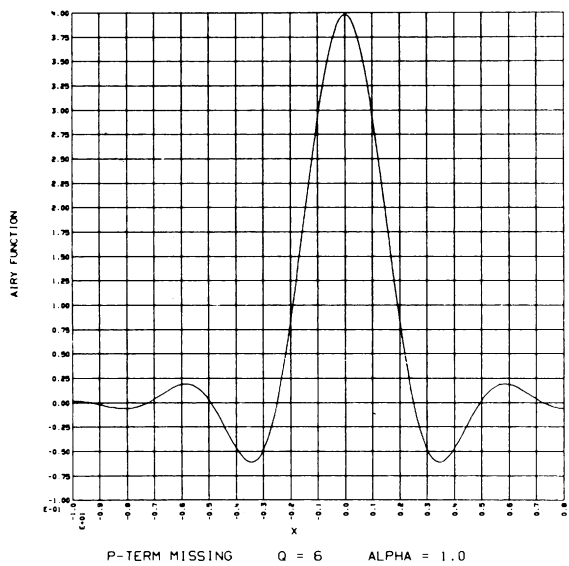


Figure 23









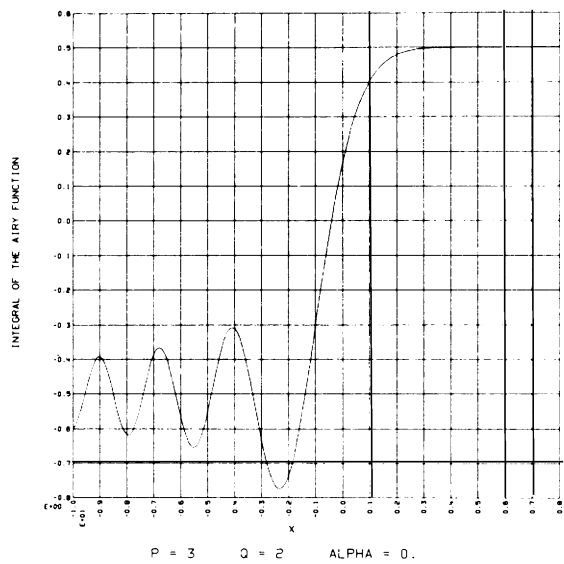


Figure 24

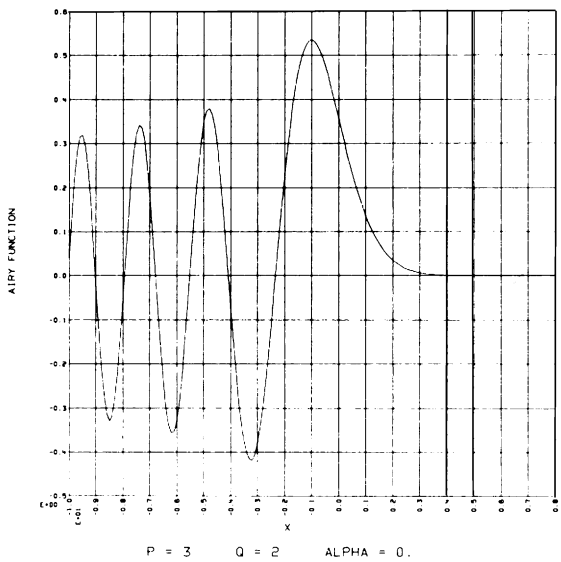


Figure 25

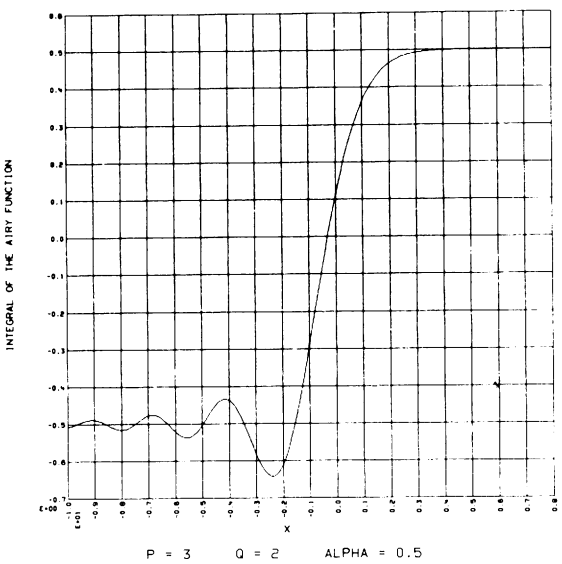


Figure 26

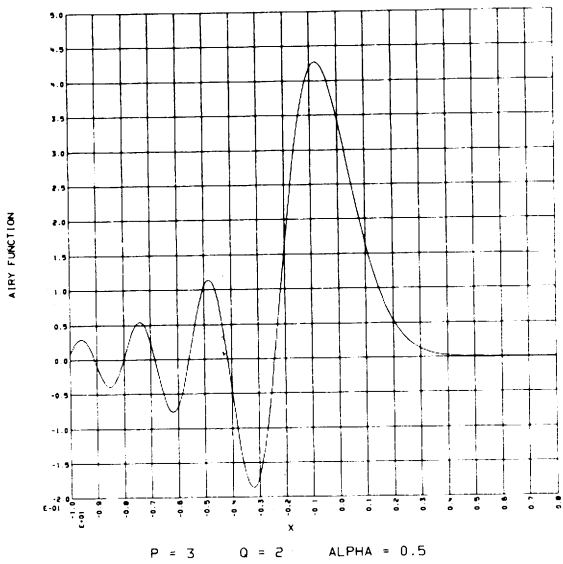


Figure 27

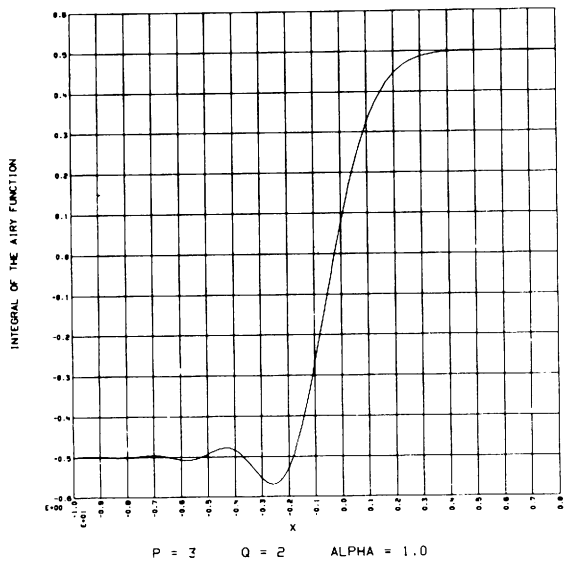


Figure 28



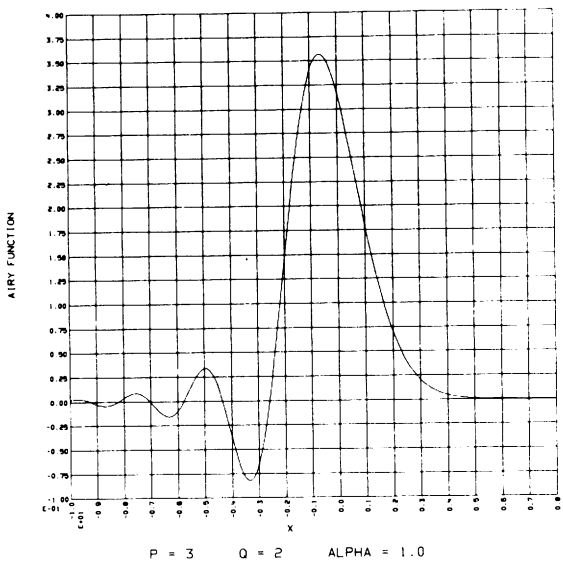


Figure 29

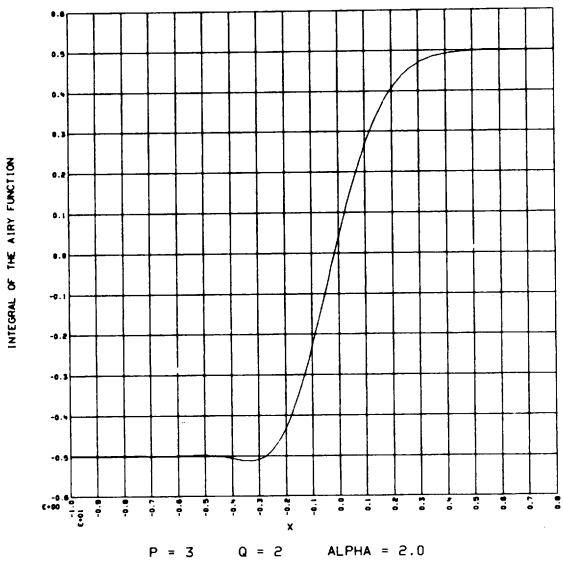


Figure 30

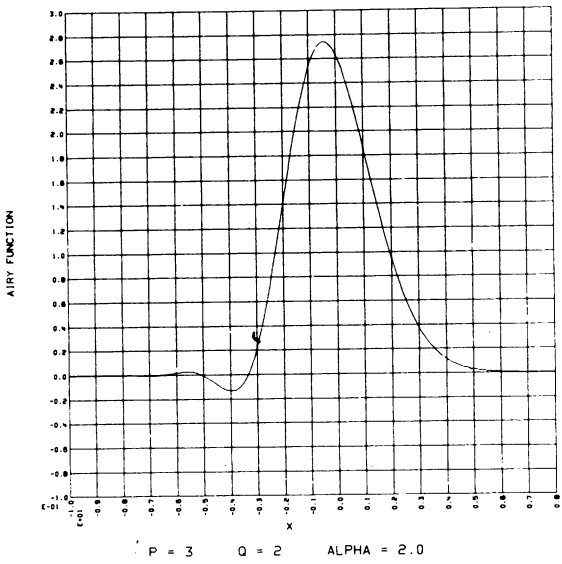


Figure 31

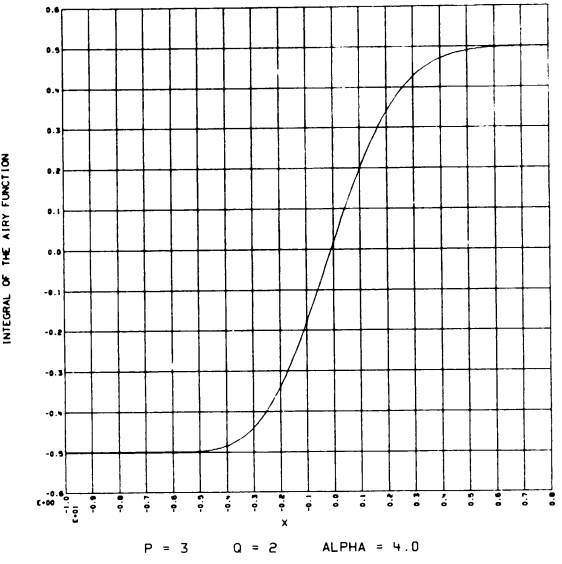


Figure 32

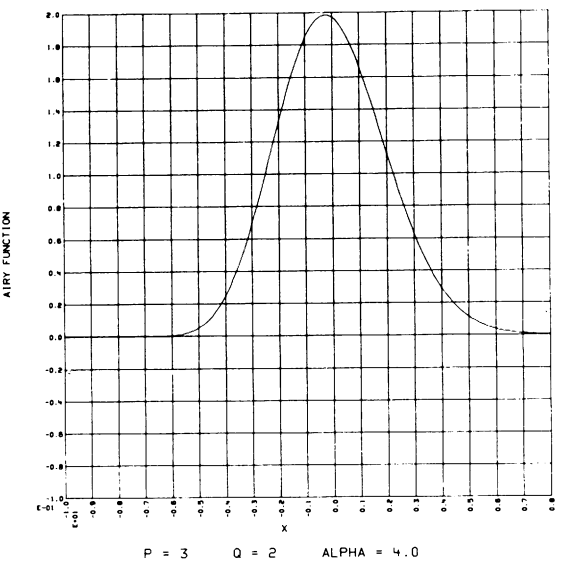


Figure 33



Table 14

P - 3 Q - 2 ALPHA - C 5

Table with columns A11101 through A21101 and rows of numerical data.





Table 16

P = 3    Q = 2    ALPHA = 7.0

K	ALIKY	ALIKY	K	ALIKY	ALIKY	K	ALIKY	ALIKY	K	ALIKY	ALIKY		
0	0	0.00000	0	0.00003	-5.5	0	0.00030	0.002326	0	0	0.270013	0	0.270000
0	0	0.00002	0	0.00012	-5.4	0	0.00010	0.002700	-0.9	0	0.000949	0	0.000001
0	0	0.00001	0	0.00003	-5.3	0	0.00025	0.001620	-0.8	0	0.170010	0	0.000703
0	0	0.00000	0	0.00001	-5.2	0	0.00000	0.000960	-0.7	0	0.147331	0	0.000606
0	0	0.00001	0	0.00002	-5.1	0	0.00020	0.001115	-0.6	0	0.226027	0	0.272330
0	0	0.00009	0	0.00008	-5.0	0	0.00070	0.002002	-0.5	0	0.002311	0	0.273710
0	0	0.00010	0	0.00009	-4.9	0	0.00131	0.002130	-0.4	0	0.007000	0	0.273018
0	0	0.00019	0	0.00050	-4.8	0	0.00112	0.002002	-0.3	0	0.040001	0	0.270000
0	0	0.00020	0	0.00043	-4.7	0	0.00130	0.002000	-0.2	0	0.013000	0	0.270000
0	0	0.00028	0	0.00031	-4.6	0	0.00113	0.001970	-0.1	0	0.010001	0	0.007000
0	0	0.00020	0	0.00010	-4.5	0	0.00118	0.001820	0	0	0.001350	0	0.001000
0	0	0.00020	0	0.00000	-4.4	0	0.00117	0.001670	0	0	0.000230	0	0.000100
0	0	0.00020	0	0.00000	-4.3	0	0.002771	0.011025	0.2	0	0.017000	0	0.250010
0	0	0.00018	0	0.00010	-4.2	0	0.002730	0.011000	0.1	0	0.002730	0	0.250010
0	0	0.00010	0	0.00010	-4.1	0	0.004002	0.012053	0	0	0.100010	0	0.230000
0	0	0.00000	0	0.00100	-4.0	0	0.00132	0.012010	0.5	0	0.001000	0	0.230000
0	0	0.00000	0	0.00100	-3.9	0	0.002000	0.013000	0	0	0.000000	0	0.210000
0	0	0.00000	0	0.00100	-3.8	0	0.002000	0.013000	0.7	0	0.001000	0	0.210000
0	0	0.00002	0	0.00100	-3.7	0	0.002000	0.013000	0	0	0.220000	0	0.200000
0	0	0.00030	0	0.00100	-3.6	0	0.002000	0.013000	0	0	0.200000	0	0.190000
0	0	0.00020	0	0.00100	-3.5	0	0.002000	0.013000	1.0	0	0.200000	0	0.180000
0	0	0.00000	0	0.00100	-3.4	0	0.002000	0.013000	1.1	0	0.000000	0	0.170000
0	0	0.00000	0	0.00100	-3.3	0	0.002000	0.013000	1.2	0	0.001000	0	0.160000
0	0	0.00000	0	0.00100	-3.2	0	0.002000	0.013000	1.3	0	0.001000	0	0.150000
0	0	0.00000	0	0.00100	-3.1	0	0.002000	0.013000	1.4	0	0.001000	0	0.140000
0	0	0.00000	0	0.00100	-3.0	0	0.002000	0.013000	1.5	0	0.001000	0	0.130000
0	0	0.00000	0	0.00100	-2.9	0	0.002000	0.013000	1.6	0	0.001000	0	0.120000
0	0	0.00000	0	0.00100	-2.8	0	0.002000	0.013000	1.7	0	0.001000	0	0.110000
0	0	0.00000	0	0.00100	-2.7	0	0.002000	0.013000	1.8	0	0.001000	0	0.100000
0	0	0.00000	0	0.00100	-2.6	0	0.002000	0.013000	1.9	0	0.001000	0	0.090000
0	0	0.00000	0	0.00100	-2.5	0	0.002000	0.013000	2.0	0	0.001000	0	0.080000
0	0	0.00000	0	0.00100	-2.4	0	0.002000	0.013000	2.1	0	0.001000	0	0.070000
0	0	0.00000	0	0.00100	-2.3	0	0.002000	0.013000	2.2	0	0.001000	0	0.060000
0	0	0.00000	0	0.00100	-2.2	0	0.002000	0.013000	2.3	0	0.001000	0	0.050000
0	0	0.00000	0	0.00100	-2.1	0	0.002000	0.013000	2.4	0	0.001000	0	0.040000
0	0	0.00000	0	0.00100	-2.0	0	0.002000	0.013000	2.5	0	0.001000	0	0.030000
0	0	0.00000	0	0.00100	-1.9	0	0.002000	0.013000	2.6	0	0.001000	0	0.020000
0	0	0.00000	0	0.00100	-1.8	0	0.002000	0.013000	2.7	0	0.001000	0	0.010000
0	0	0.00000	0	0.00100	-1.7	0	0.002000	0.013000	2.8	0	0.001000	0	0.000000
0	0	0.00000	0	0.00100	-1.6	0	0.002000	0.013000	2.9	0	0.001000	0	0.000000
0	0	0.00000	0	0.00100	-1.5	0	0.002000	0.013000	3.0	0	0.001000	0	0.000000
0	0	0.00000	0	0.00100	-1.4	0	0.002000	0.013000	3.1	0	0.001000	0	0.000000
0	0	0.00000	0	0.00100	-1.3	0	0.002000	0.013000	3.2	0	0.001000	0	0.000000
0	0	0.00000	0	0.00100	-1.2	0	0.002000	0.013000	3.3	0	0.001000	0	0.000000
0	0	0.00000	0	0.00100	-1.1	0	0.002000	0.013000	3.4	0	0.001000	0	0.000000
0	0	0.00000	0	0.00100	-1.0	0	0.002000	0.013000	3.5	0	0.001000	0	0.000000

Table 17

P = 3 Q = 2 ALPHA = 4.0

X	AI1X1	AI1X1	X	AI1X1	AI1X1	X	AI1X1	AI1X1	X	AI1X1	AI1X1
1.0	0	0	0	0	0	1.0	0	0	0	0	0
0.9	0	0	0	0	0	0.9	0	0	0	0	0
0.8	0	0	0	0	0	0.8	0	0	0	0	0
0.7	0	0	0	0	0	0.7	0	0	0	0	0
0.6	0	0	0	0	0	0.6	0	0	0	0	0
0.5	0	0	0	0	0	0.5	0	0	0	0	0
0.4	0	0	0	0	0	0.4	0	0	0	0	0
0.3	0	0	0	0	0	0.3	0	0	0	0	0
0.2	0	0	0	0	0	0.2	0	0	0	0	0
0.1	0	0	0	0	0	0.1	0	0	0	0	0
0.0	0	0	0	0	0	0.0	0	0	0	0	0
-0.1	0	0	0	0	0	-0.1	0	0	0	0	0
-0.2	0	0	0	0	0	-0.2	0	0	0	0	0
-0.3	0	0	0	0	0	-0.3	0	0	0	0	0
-0.4	0	0	0	0	0	-0.4	0	0	0	0	0
-0.5	0	0	0	0	0	-0.5	0	0	0	0	0
-0.6	0	0	0	0	0	-0.6	0	0	0	0	0
-0.7	0	0	0	0	0	-0.7	0	0	0	0	0
-0.8	0	0	0	0	0	-0.8	0	0	0	0	0
-0.9	0	0	0	0	0	-0.9	0	0	0	0	0
-1.0	0	0	0	0	0	-1.0	0	0	0	0	0

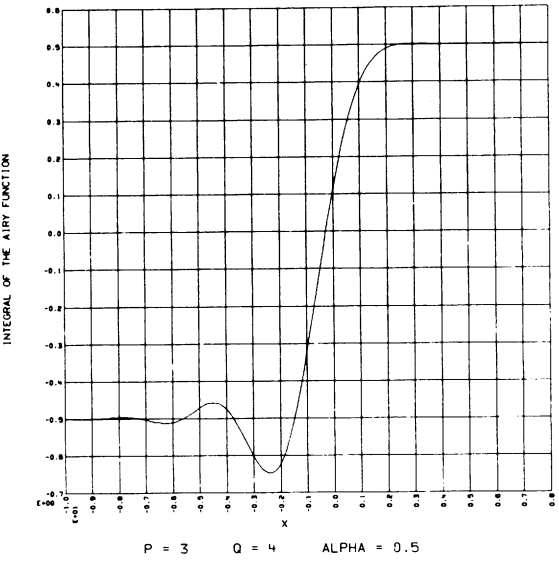


Figure 34

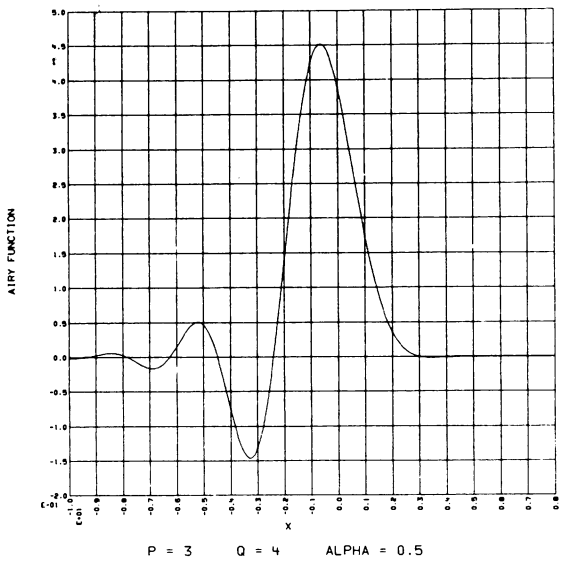


Figure 35

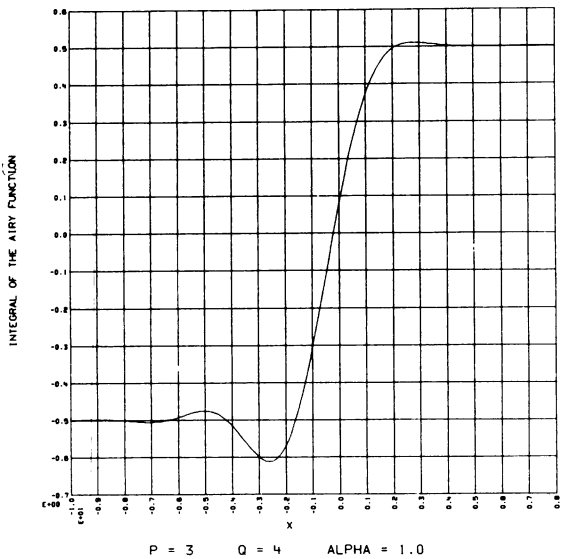


Figure 36

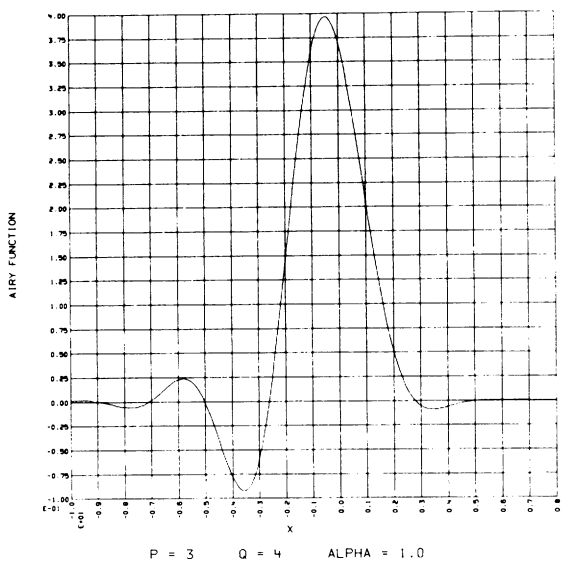


Figure 37

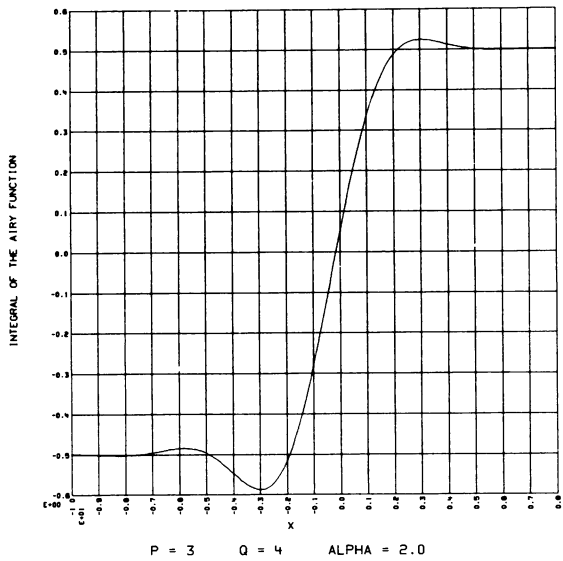


Figure 38

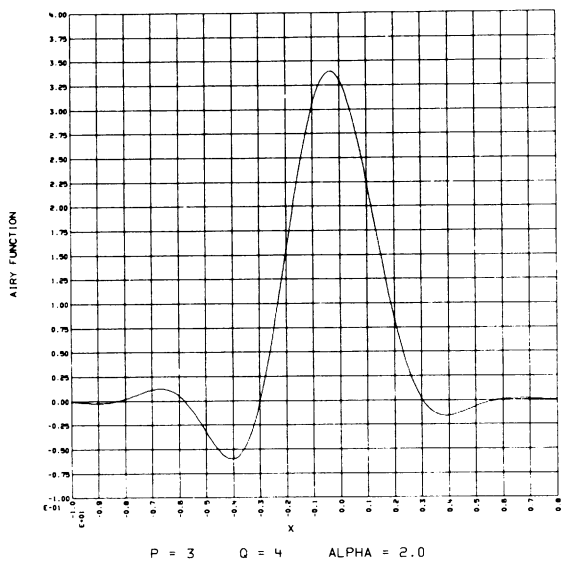


Figure 39



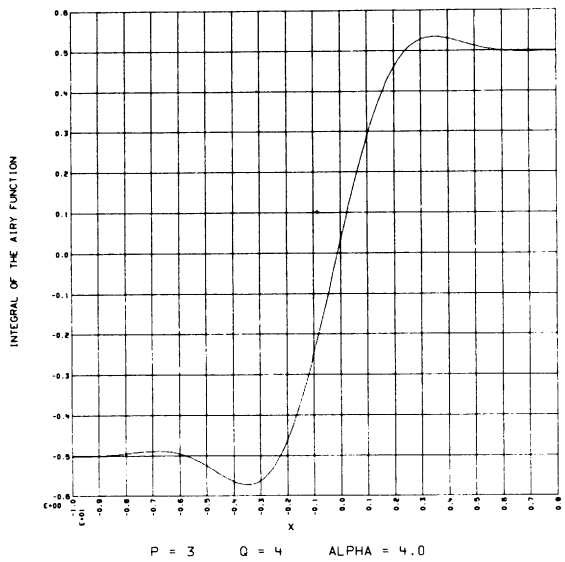


Figure 40

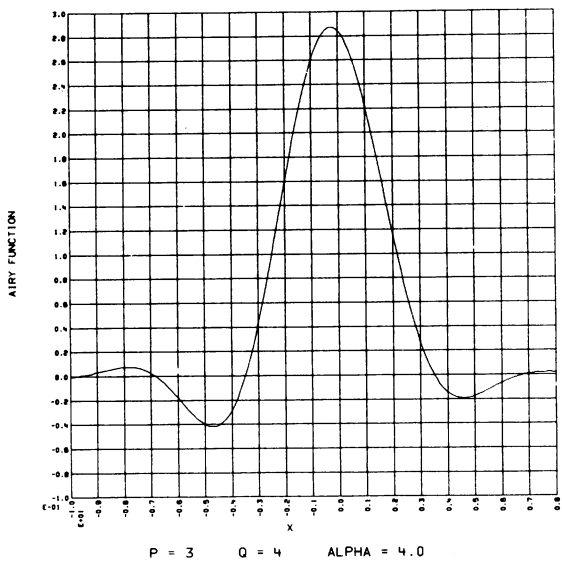


Figure 41









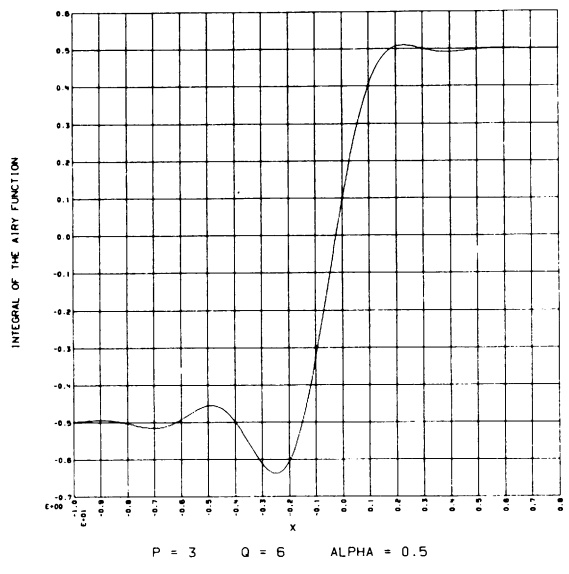


Figure 42

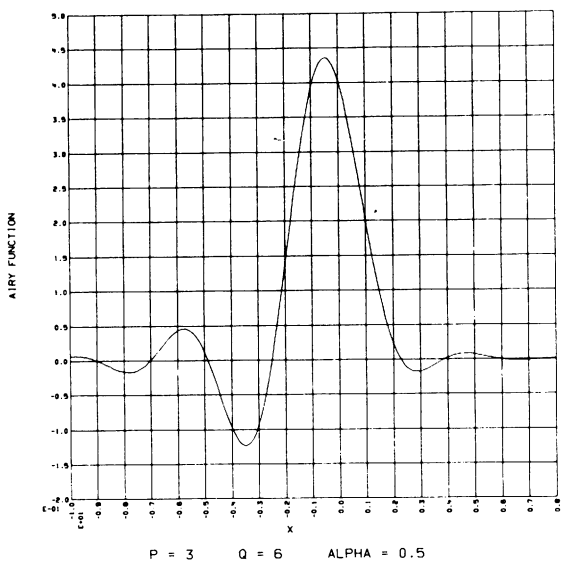


Figure 43



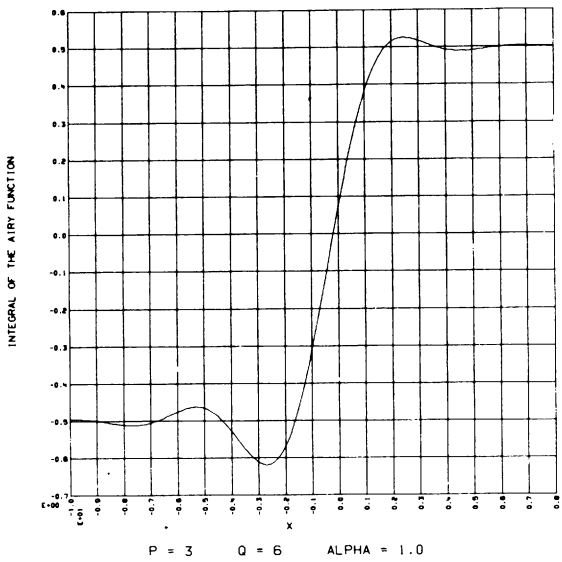


Figure 44

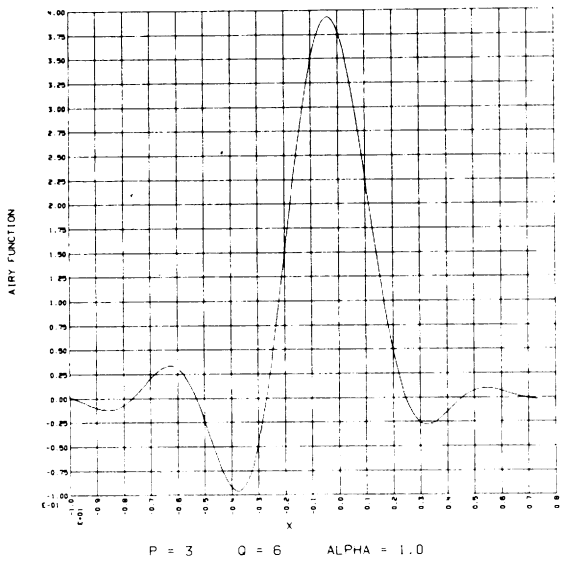


Figure 45

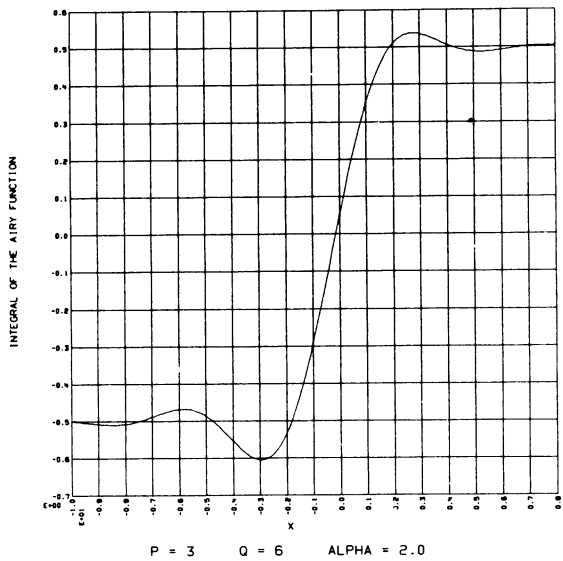


Figure 46

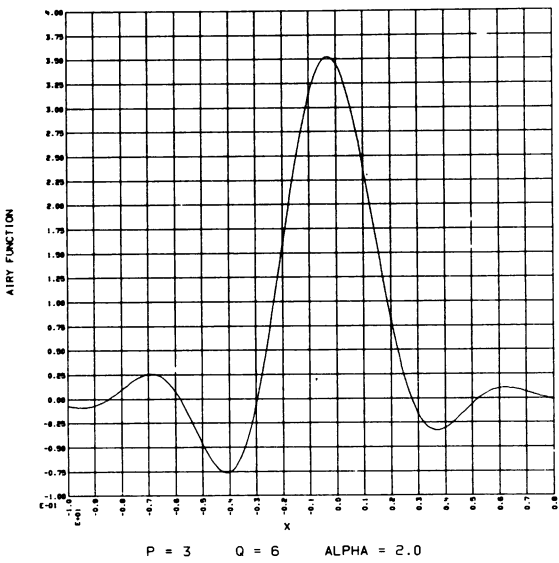


Figure 47

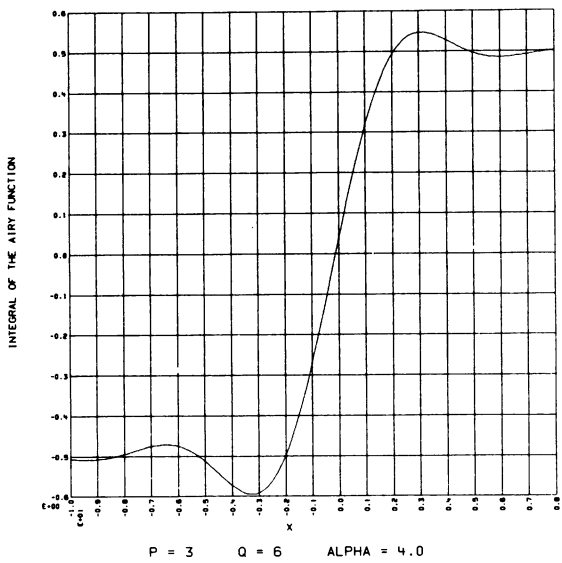


Figure 48

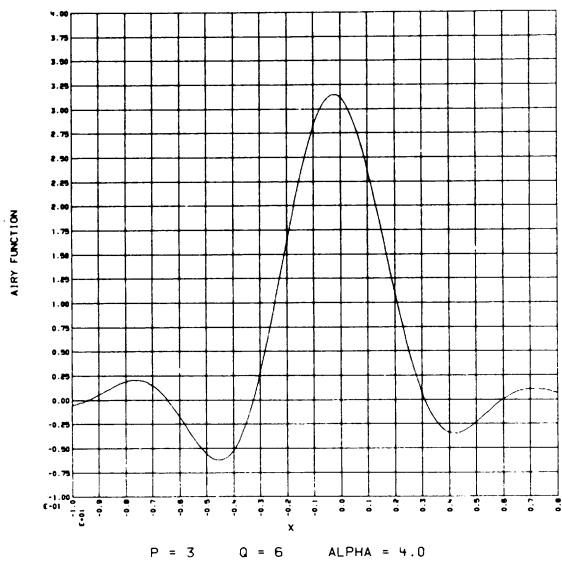


Figure 49







Table 24

P = 3 O = 6 ALPHA = P 0

X	ALPHA	ALPHA	X	ALPHA	ALPHA	X	ALPHA	ALPHA	X	ALPHA	ALPHA
0.0	0.00000	0.00000	0.0	0.71336	-0.07999	-1.0	0.00000	0.00000	0.0	0.71336	0.00000
0.1	0.00000	0.00000	0.1	0.71336	-0.07999	-1.0	0.00000	0.00000	0.1	0.71336	0.00000
0.2	0.00000	0.00000	0.2	0.71336	-0.07999	-1.0	0.00000	0.00000	0.2	0.71336	0.00000
0.3	0.00000	0.00000	0.3	0.71336	-0.07999	-1.0	0.00000	0.00000	0.3	0.71336	0.00000
0.4	0.00000	0.00000	0.4	0.71336	-0.07999	-1.0	0.00000	0.00000	0.4	0.71336	0.00000
0.5	0.00000	0.00000	0.5	0.71336	-0.07999	-1.0	0.00000	0.00000	0.5	0.71336	0.00000
0.6	0.00000	0.00000	0.6	0.71336	-0.07999	-1.0	0.00000	0.00000	0.6	0.71336	0.00000
0.7	0.00000	0.00000	0.7	0.71336	-0.07999	-1.0	0.00000	0.00000	0.7	0.71336	0.00000
0.8	0.00000	0.00000	0.8	0.71336	-0.07999	-1.0	0.00000	0.00000	0.8	0.71336	0.00000
0.9	0.00000	0.00000	0.9	0.71336	-0.07999	-1.0	0.00000	0.00000	0.9	0.71336	0.00000
1.0	0.00000	0.00000	1.0	0.71336	-0.07999	-1.0	0.00000	0.00000	1.0	0.71336	0.00000



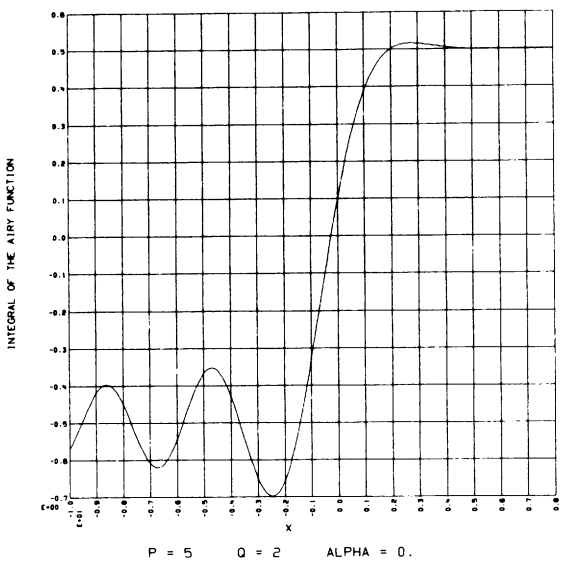
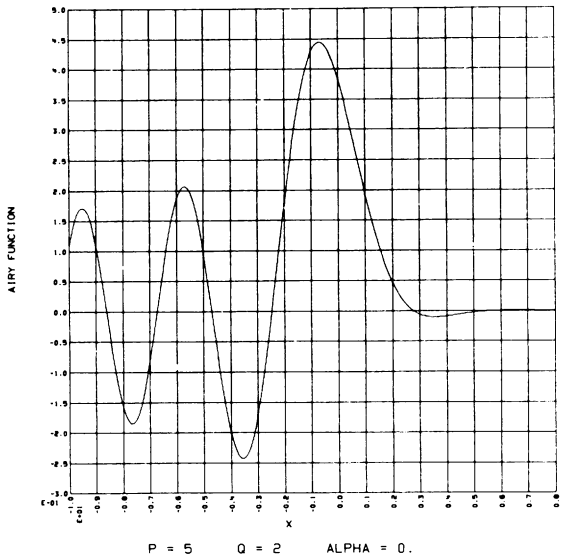


Figure 50



P = 5    Q = 2    ALPHA = 0.

Figure 51

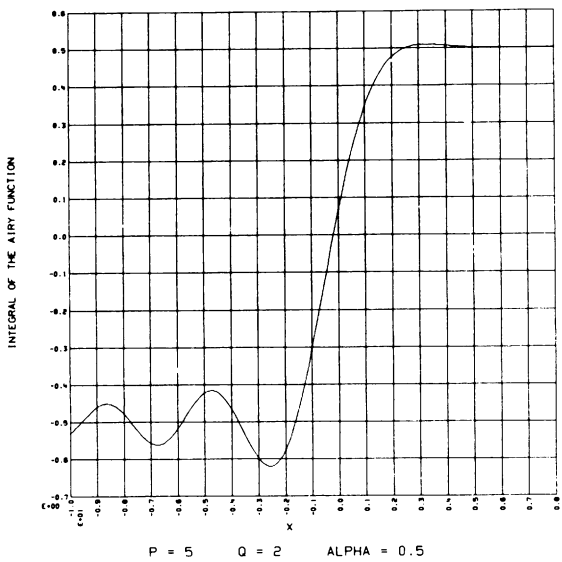


Figure 52

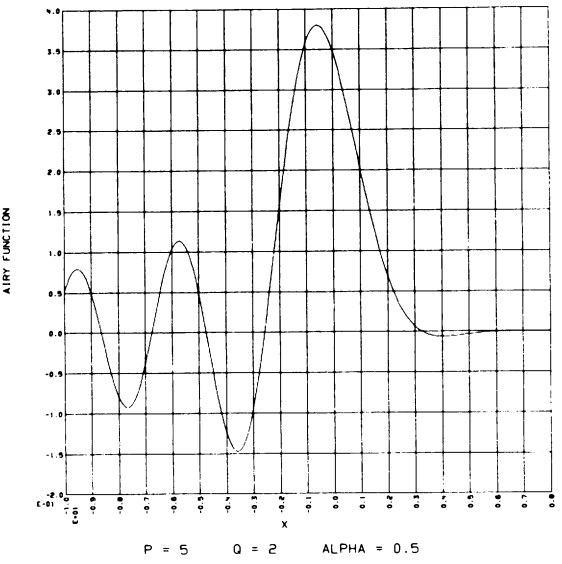


Figure 53

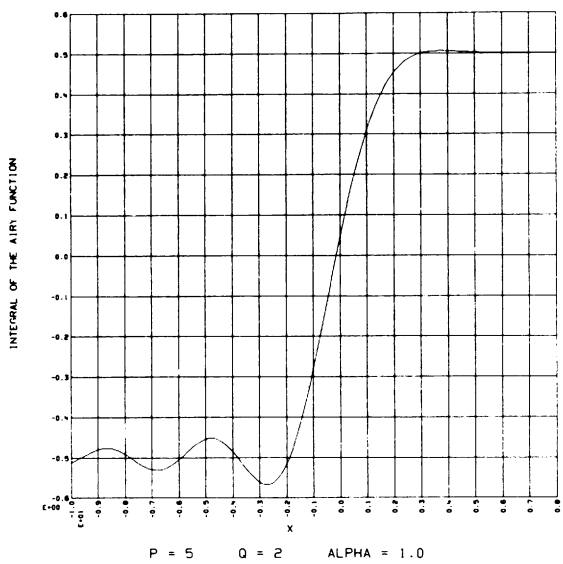


Figure 54

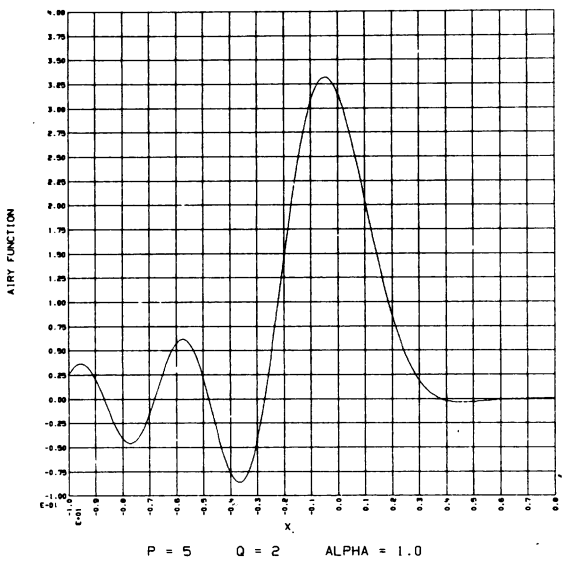


Figure 55



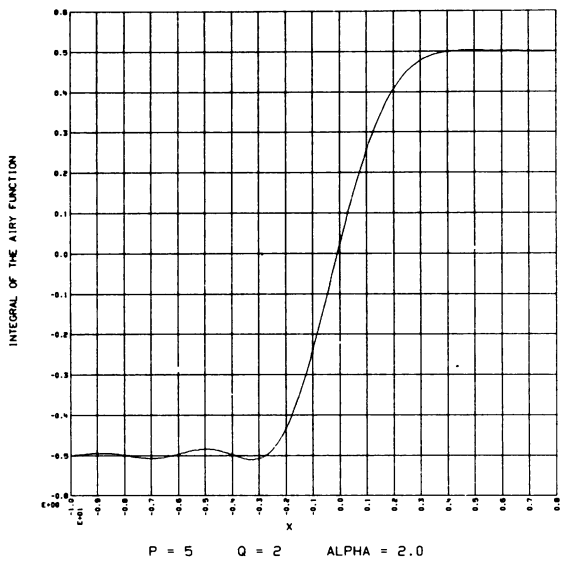


Figure 56

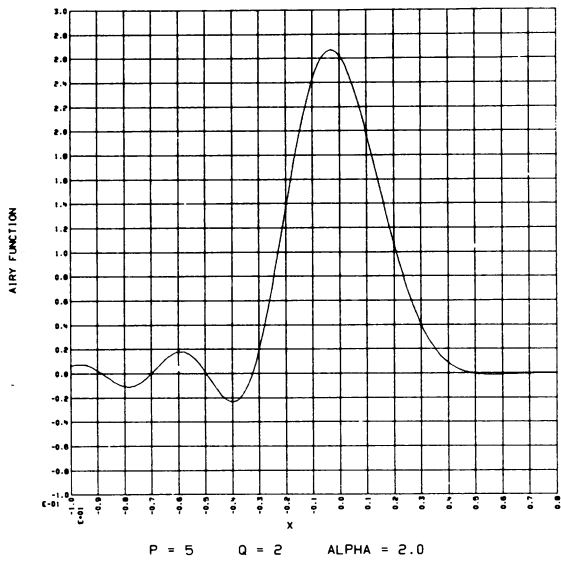


Figure 57

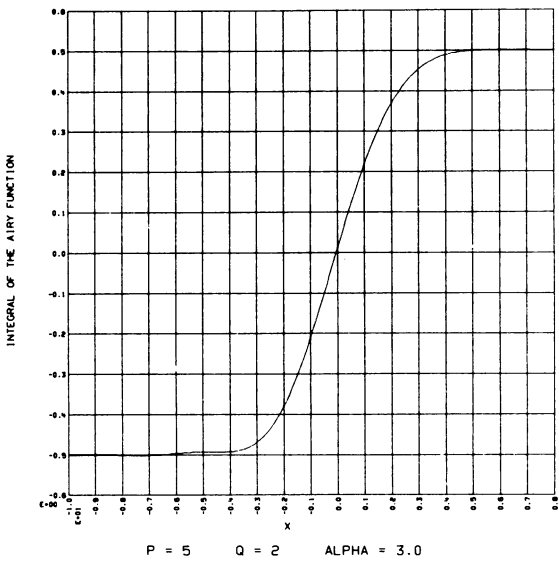


Figure 58

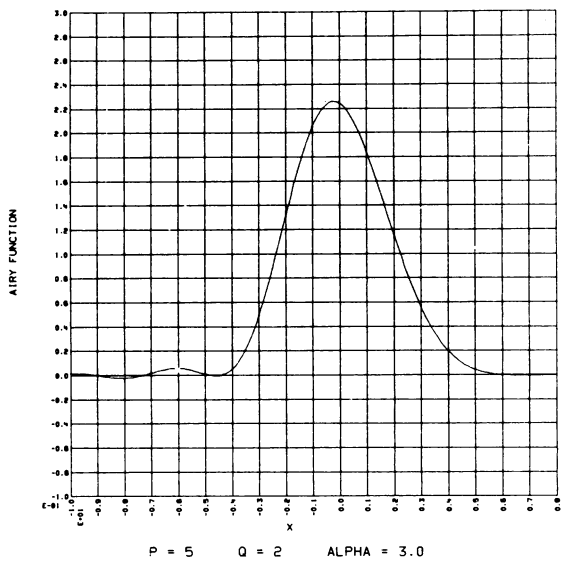


Figure 59

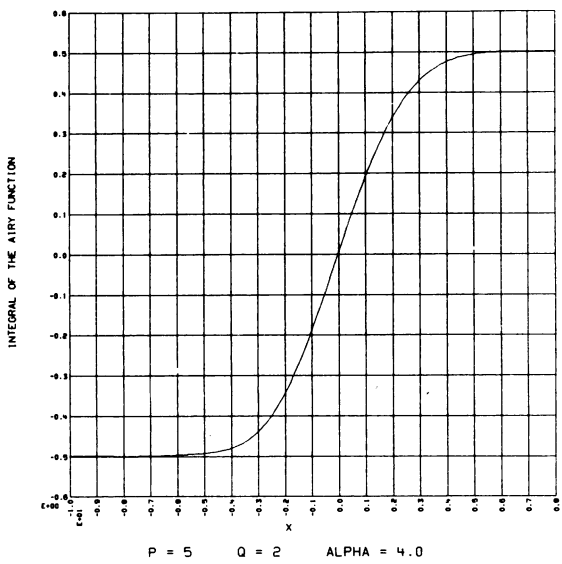


Figure 60

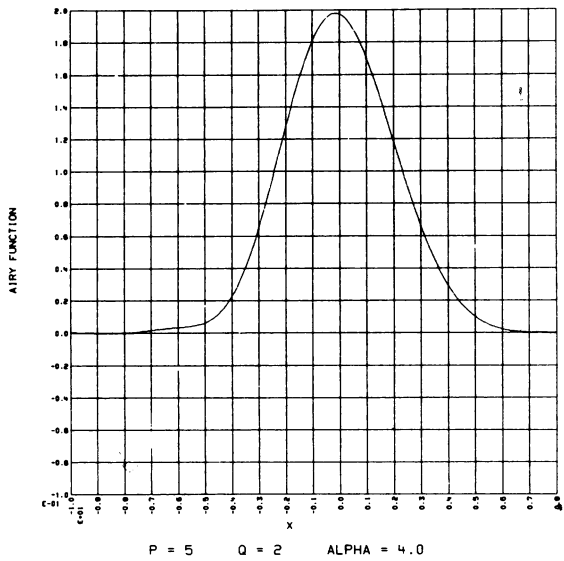


Figure 61







Table 28

P = 5 Q = 2 ALPHA = 1.0

X	ALL18	ALL18	X	ALL18	ALL18	X	ALL18	ALL18	X	ALL18	ALL18	
10	0	0.511874	0	0.029422	5	0	0.471582	0.006631	-1	0	0.270927	0.000970
9	0	0.500312	0	0.009008	5	0	0.460763	0.011880	0	0	0.277790	0.018370
8	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
7	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
6	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
5	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
4	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
3	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
2	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
1	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000

10	0	0.511874	0	0.029422	5	0	0.471582	0.006631	-1	0	0.270927	0.000970
9	0	0.500312	0	0.009008	5	0	0.460763	0.011880	0	0	0.277790	0.018370
8	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
7	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
6	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
5	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
4	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
3	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
2	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
1	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000

10	0	0.511874	0	0.029422	5	0	0.471582	0.006631	-1	0	0.270927	0.000970
9	0	0.500312	0	0.009008	5	0	0.460763	0.011880	0	0	0.277790	0.018370
8	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
7	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
6	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
5	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
4	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
3	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
2	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000
1	0	0.500000	0	0.000000	5	0	0.460000	0.000000	0	0	0.278000	0.020000

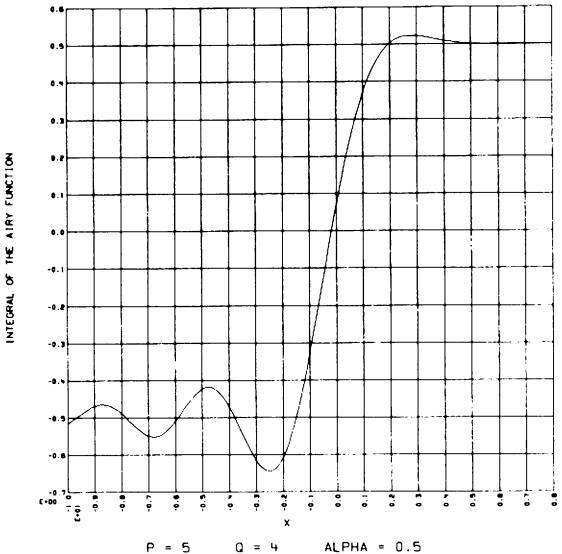


Table 30

p = 5 q = 2 ALPHA = 3.0

K	ALPHA	ALIX	ALIX	Y	ALPHA	ALIX	X	ALPHA	ALIX	X	ALPHA	ALIX	ALIX
10.0	0	0.0001	0.001468	-5.5	0	0.0001	1.0	0	0.0001	1.0	0	0.0001	0.001468
9.9	0	0.0010	0.001188	-5.4	0	0.0022	0.0	0	0.0010	1.0	0	0.0010	0.001188
9.8	0	0.0025	0.001708	-5.3	0	0.0038	0.0	0	0.0025	1.0	0	0.0025	0.001708
9.7	0	0.0050	0.002127	-5.2	0	0.0052	0.0	0	0.0050	1.0	0	0.0050	0.002127
9.6	0	0.0075	0.002412	-5.1	0	0.0062	0.0	0	0.0075	1.0	0	0.0075	0.002412
9.5	0	0.0100	0.002588	-5.0	0	0.0070	0.0	0	0.0100	1.0	0	0.0100	0.002588
9.4	0	0.0125	0.002684	-4.9	0	0.0077	0.0	0	0.0125	1.0	0	0.0125	0.002684
9.3	0	0.0150	0.002711	-4.8	0	0.0082	0.0	0	0.0150	1.0	0	0.0150	0.002711
9.2	0	0.0175	0.002680	-4.7	0	0.0086	0.0	0	0.0175	1.0	0	0.0175	0.002680
9.1	0	0.0200	0.002584	-4.6	0	0.0089	0.0	0	0.0200	1.0	0	0.0200	0.002584
9.0	0	0.0225	0.002384	-4.5	0	0.0092	0.0	0	0.0225	1.0	0	0.0225	0.002384
8.9	0	0.0250	0.002116	-4.4	0	0.0094	0.0	0	0.0250	1.0	0	0.0250	0.002116
8.8	0	0.0275	0.001784	-4.3	0	0.0096	0.0	0	0.0275	1.0	0	0.0275	0.001784
8.7	0	0.0300	0.001397	-4.2	0	0.0097	0.0	0	0.0300	1.0	0	0.0300	0.001397
8.6	0	0.0325	0.000963	-4.1	0	0.0098	0.0	0	0.0325	1.0	0	0.0325	0.000963
8.5	0	0.0350	0.000489	-4.0	0	0.0099	0.0	0	0.0350	1.0	0	0.0350	0.000489
8.4	0	0.0375	0.000075	-3.9	0	0.0100	0.0	0	0.0375	1.0	0	0.0375	0.000075
8.3	0	0.0400	0.000000	-3.8	0	0.0100	0.0	0	0.0400	1.0	0	0.0400	0.000000
8.2	0	0.0425	0.000000	-3.7	0	0.0100	0.0	0	0.0425	1.0	0	0.0425	0.000000
8.1	0	0.0450	0.000000	-3.6	0	0.0100	0.0	0	0.0450	1.0	0	0.0450	0.000000
8.0	0	0.0475	0.000000	-3.5	0	0.0100	0.0	0	0.0475	1.0	0	0.0475	0.000000
7.9	0	0.0500	0.000000	-3.4	0	0.0100	0.0	0	0.0500	1.0	0	0.0500	0.000000
7.8	0	0.0525	0.000000	-3.3	0	0.0100	0.0	0	0.0525	1.0	0	0.0525	0.000000
7.7	0	0.0550	0.000000	-3.2	0	0.0100	0.0	0	0.0550	1.0	0	0.0550	0.000000
7.6	0	0.0575	0.000000	-3.1	0	0.0100	0.0	0	0.0575	1.0	0	0.0575	0.000000
7.5	0	0.0600	0.000000	-3.0	0	0.0100	0.0	0	0.0600	1.0	0	0.0600	0.000000
7.4	0	0.0625	0.000000	-2.9	0	0.0100	0.0	0	0.0625	1.0	0	0.0625	0.000000
7.3	0	0.0650	0.000000	-2.8	0	0.0100	0.0	0	0.0650	1.0	0	0.0650	0.000000
7.2	0	0.0675	0.000000	-2.7	0	0.0100	0.0	0	0.0675	1.0	0	0.0675	0.000000
7.1	0	0.0700	0.000000	-2.6	0	0.0100	0.0	0	0.0700	1.0	0	0.0700	0.000000
7.0	0	0.0725	0.000000	-2.5	0	0.0100	0.0	0	0.0725	1.0	0	0.0725	0.000000
6.9	0	0.0750	0.000000	-2.4	0	0.0100	0.0	0	0.0750	1.0	0	0.0750	0.000000
6.8	0	0.0775	0.000000	-2.3	0	0.0100	0.0	0	0.0775	1.0	0	0.0775	0.000000
6.7	0	0.0800	0.000000	-2.2	0	0.0100	0.0	0	0.0800	1.0	0	0.0800	0.000000
6.6	0	0.0825	0.000000	-2.1	0	0.0100	0.0	0	0.0825	1.0	0	0.0825	0.000000
6.5	0	0.0850	0.000000	-2.0	0	0.0100	0.0	0	0.0850	1.0	0	0.0850	0.000000
6.4	0	0.0875	0.000000	-1.9	0	0.0100	0.0	0	0.0875	1.0	0	0.0875	0.000000
6.3	0	0.0900	0.000000	-1.8	0	0.0100	0.0	0	0.0900	1.0	0	0.0900	0.000000
6.2	0	0.0925	0.000000	-1.7	0	0.0100	0.0	0	0.0925	1.0	0	0.0925	0.000000
6.1	0	0.0950	0.000000	-1.6	0	0.0100	0.0	0	0.0950	1.0	0	0.0950	0.000000
6.0	0	0.0975	0.000000	-1.5	0	0.0100	0.0	0	0.0975	1.0	0	0.0975	0.000000
5.9	0	0.1000	0.000000	-1.4	0	0.0100	0.0	0	0.1000	1.0	0	0.1000	0.000000
5.8	0	0.1025	0.000000	-1.3	0	0.0100	0.0	0	0.1025	1.0	0	0.1025	0.000000
5.7	0	0.1050	0.000000	-1.2	0	0.0100	0.0	0	0.1050	1.0	0	0.1050	0.000000
5.6	0	0.1075	0.000000	-1.1	0	0.0100	0.0	0	0.1075	1.0	0	0.1075	0.000000
5.5	0	0.1100	0.000000	-1.0	0	0.0100	0.0	0	0.1100	1.0	0	0.1100	0.000000
5.4	0	0.1125	0.000000	-0.9	0	0.0100	0.0	0	0.1125	1.0	0	0.1125	0.000000





P = 5    Q = 4    ALPHA = 0.5

Figure 62

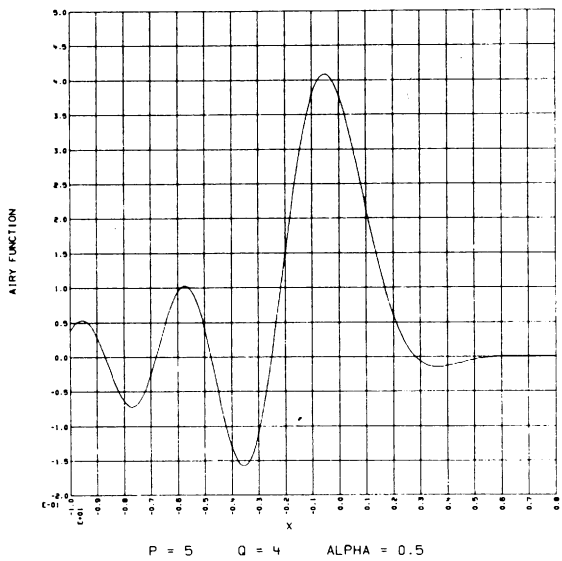


Figure 63

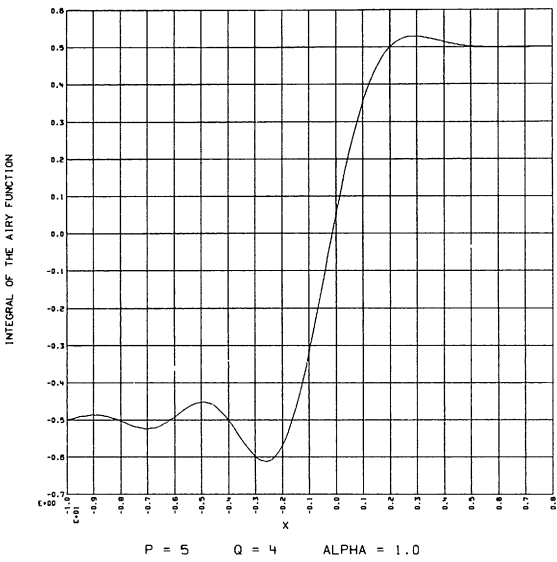


Figure 64

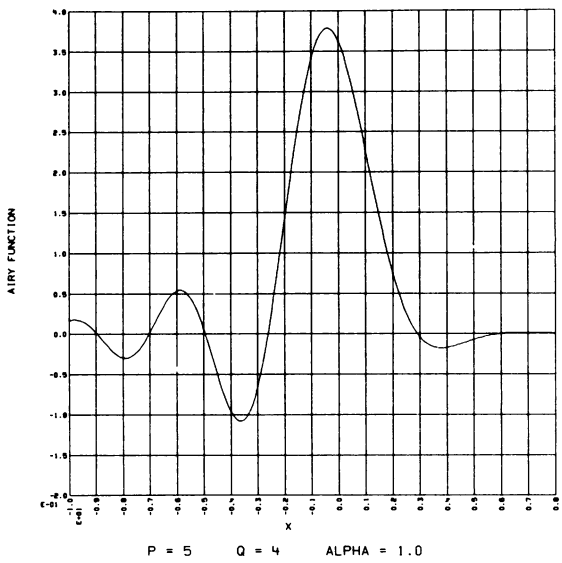


Figure 65



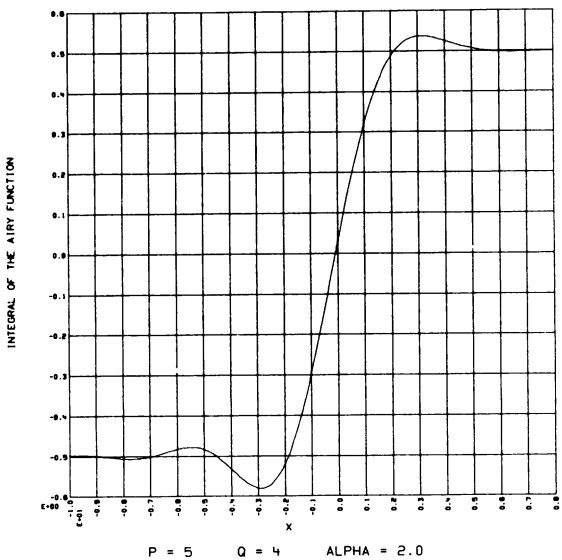


Figure 66

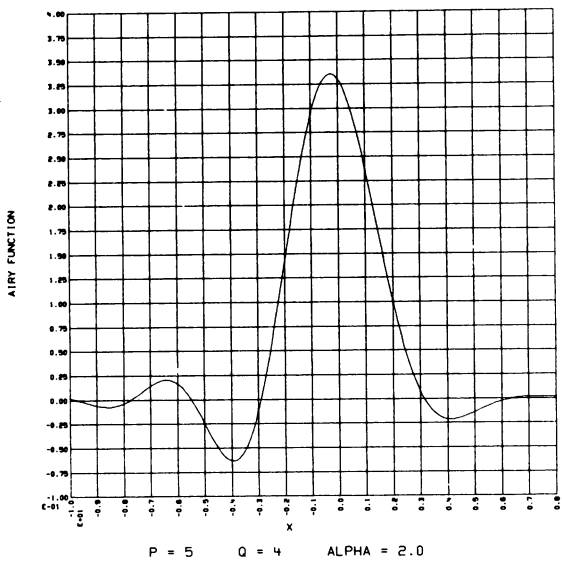


Figure 67

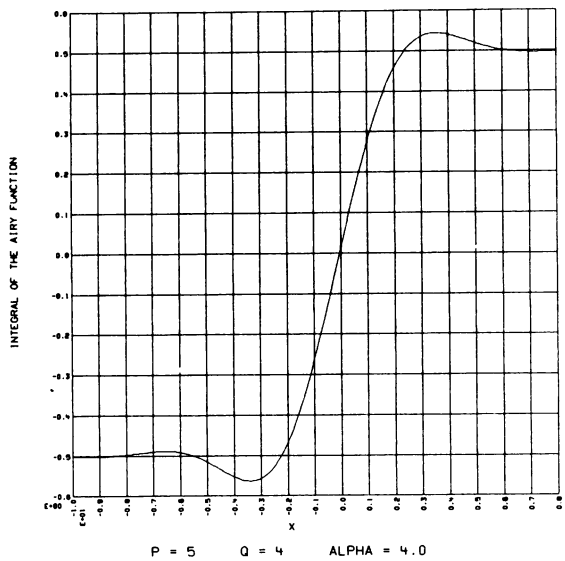


Figure 68

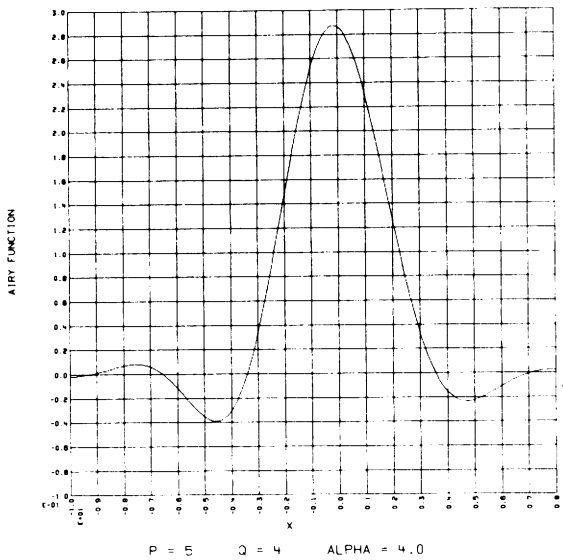


Figure 69





Table 34

P = 5 Q = 4 ALPHA = 2.0

X	ALPHA	ALPHA	3	ALPHA	ALPHA	5	ALPHA	ALPHA	7	ALPHA	ALPHA
-10.0	0.000000	0.001363	-5.5	-0.478601	0.000000	-1.0	-0.000000	0.000000	3.0	0.000000	-0.010000
-9.0	0.000000	0.000703	-5.4	-0.478710	0.000000	-0.9	-0.000000	0.000000	3.0	0.000000	-0.010000
-8.0	0.000000	0.000387	-5.3	-0.479008	0.000000	-0.8	-0.000000	0.000000	3.0	0.000000	-0.010000
-7.0	0.000000	0.000204	-5.2	-0.480000	0.000000	-0.7	-0.000000	0.000000	3.0	0.000000	-0.010000
-6.0	0.000000	0.000129	-5.1	-0.482000	0.000000	-0.6	-0.000000	0.000000	3.0	0.000000	-0.010000
-5.0	0.000000	0.000078	-5.0	-0.485000	0.000000	-0.5	-0.000000	0.000000	3.0	0.000000	-0.010000
-4.0	0.000000	0.000048	-4.9	-0.489000	0.000000	-0.4	-0.000000	0.000000	3.0	0.000000	-0.010000
-3.0	0.000000	0.000029	-4.8	-0.494000	0.000000	-0.3	-0.000000	0.000000	3.0	0.000000	-0.010000
-2.0	0.000000	0.000018	-4.7	-0.500000	0.000000	-0.2	-0.000000	0.000000	3.0	0.000000	-0.010000
-1.0	0.000000	0.000011	-4.6	-0.507000	0.000000	-0.1	-0.000000	0.000000	3.0	0.000000	-0.010000
0.0	0.000000	0.000007	-4.5	-0.515000	0.000000	0.0	0.000000	0.000000	3.0	0.000000	-0.010000
1.0	0.000000	0.000004	-4.4	-0.524000	0.000000	0.1	0.000000	0.000000	3.0	0.000000	-0.010000
2.0	0.000000	0.000003	-4.3	-0.534000	0.000000	0.2	0.000000	0.000000	3.0	0.000000	-0.010000
3.0	0.000000	0.000002	-4.2	-0.545000	0.000000	0.3	0.000000	0.000000	3.0	0.000000	-0.010000
4.0	0.000000	0.000001	-4.1	-0.557000	0.000000	0.4	0.000000	0.000000	3.0	0.000000	-0.010000
5.0	0.000000	0.000001	-4.0	-0.570000	0.000000	0.5	0.000000	0.000000	3.0	0.000000	-0.010000
6.0	0.000000	0.000000	-3.9	-0.584000	0.000000	0.6	0.000000	0.000000	3.0	0.000000	-0.010000
7.0	0.000000	0.000000	-3.8	-0.599000	0.000000	0.7	0.000000	0.000000	3.0	0.000000	-0.010000
8.0	0.000000	0.000000	-3.7	-0.615000	0.000000	0.8	0.000000	0.000000	3.0	0.000000	-0.010000
9.0	0.000000	0.000000	-3.6	-0.632000	0.000000	0.9	0.000000	0.000000	3.0	0.000000	-0.010000
10.0	0.000000	0.000000	-3.5	-0.650000	0.000000	1.0	0.000000	0.000000	3.0	0.000000	-0.010000
11.0	0.000000	0.000000	-3.4	-0.669000	0.000000	1.1	0.000000	0.000000	3.0	0.000000	-0.010000
12.0	0.000000	0.000000	-3.3	-0.689000	0.000000	1.2	0.000000	0.000000	3.0	0.000000	-0.010000
13.0	0.000000	0.000000	-3.2	-0.710000	0.000000	1.3	0.000000	0.000000	3.0	0.000000	-0.010000
14.0	0.000000	0.000000	-3.1	-0.732000	0.000000	1.4	0.000000	0.000000	3.0	0.000000	-0.010000
15.0	0.000000	0.000000	-3.0	-0.755000	0.000000	1.5	0.000000	0.000000	3.0	0.000000	-0.010000
16.0	0.000000	0.000000	-2.9	-0.779000	0.000000	1.6	0.000000	0.000000	3.0	0.000000	-0.010000
17.0	0.000000	0.000000	-2.8	-0.804000	0.000000	1.7	0.000000	0.000000	3.0	0.000000	-0.010000
18.0	0.000000	0.000000	-2.7	-0.830000	0.000000	1.8	0.000000	0.000000	3.0	0.000000	-0.010000
19.0	0.000000	0.000000	-2.6	-0.857000	0.000000	1.9	0.000000	0.000000	3.0	0.000000	-0.010000
20.0	0.000000	0.000000	-2.5	-0.885000	0.000000	2.0	0.000000	0.000000	3.0	0.000000	-0.010000
21.0	0.000000	0.000000	-2.4	-0.914000	0.000000	2.1	0.000000	0.000000	3.0	0.000000	-0.010000
22.0	0.000000	0.000000	-2.3	-0.944000	0.000000	2.2	0.000000	0.000000	3.0	0.000000	-0.010000
23.0	0.000000	0.000000	-2.2	-0.975000	0.000000	2.3	0.000000	0.000000	3.0	0.000000	-0.010000
24.0	0.000000	0.000000	-2.1	-1.007000	0.000000	2.4	0.000000	0.000000	3.0	0.000000	-0.010000
25.0	0.000000	0.000000	-2.0	-1.040000	0.000000	2.5	0.000000	0.000000	3.0	0.000000	-0.010000
26.0	0.000000	0.000000	-1.9	-1.074000	0.000000	2.6	0.000000	0.000000	3.0	0.000000	-0.010000
27.0	0.000000	0.000000	-1.8	-1.109000	0.000000	2.7	0.000000	0.000000	3.0	0.000000	-0.010000
28.0	0.000000	0.000000	-1.7	-1.145000	0.000000	2.8	0.000000	0.000000	3.0	0.000000	-0.010000
29.0	0.000000	0.000000	-1.6	-1.182000	0.000000	2.9	0.000000	0.000000	3.0	0.000000	-0.010000
30.0	0.000000	0.000000	-1.5	-1.220000	0.000000	3.0	0.000000	0.000000	3.0	0.000000	-0.010000
31.0	0.000000	0.000000	-1.4	-1.259000	0.000000	3.1	0.000000	0.000000	3.0	0.000000	-0.010000
32.0	0.000000	0.000000	-1.3	-1.300000	0.000000	3.2	0.000000	0.000000	3.0	0.000000	-0.010000
33.0	0.000000	0.000000	-1.2	-1.342000	0.000000	3.3	0.000000	0.000000	3.0	0.000000	-0.010000
34.0	0.000000	0.000000	-1.1	-1.385000	0.000000	3.4	0.000000	0.000000	3.0	0.000000	-0.010000
35.0	0.000000	0.000000	-1.0	-1.430000	0.000000	3.5	0.000000	0.000000	3.0	0.000000	-0.010000





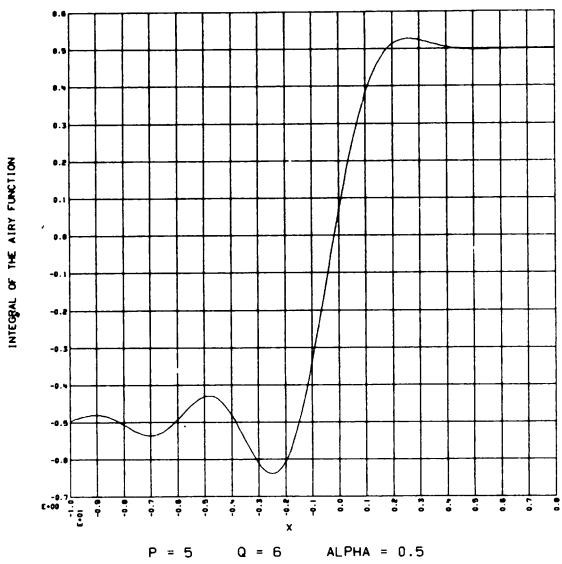


Figure 70

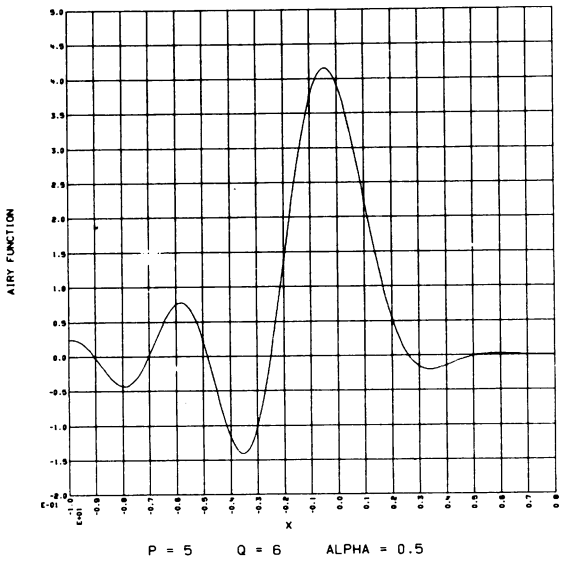


Figure 71

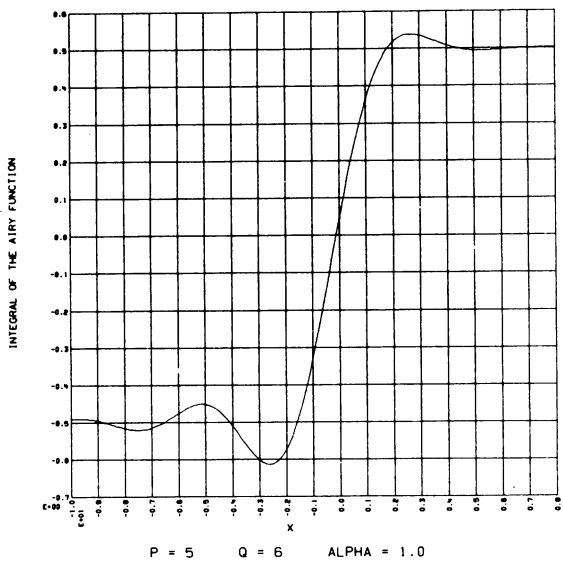


Figure 72

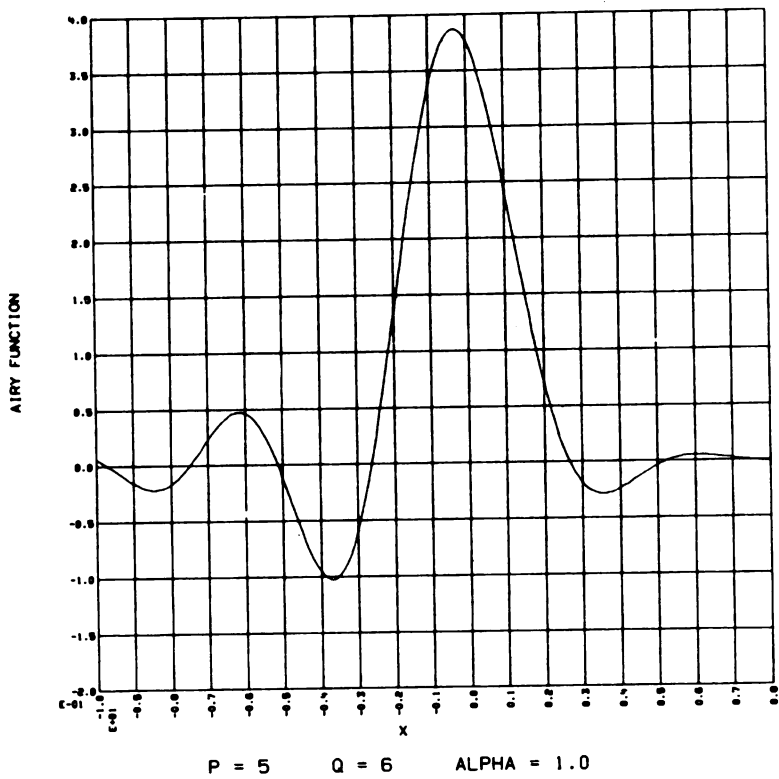
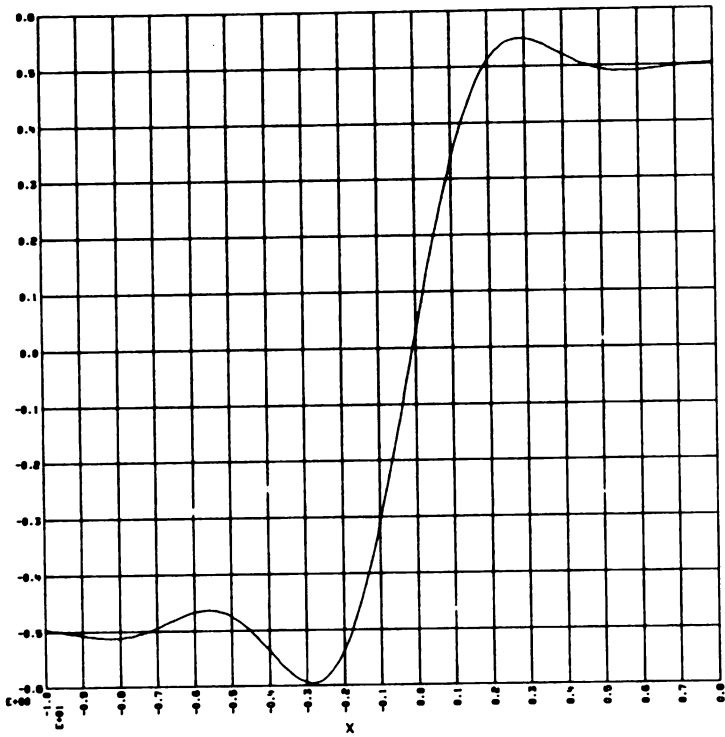


Figure 73

INTEGRAL OF THE AIRY FUNCTION



P = 5    Q = 6    ALPHA = 2.0

Figure 74

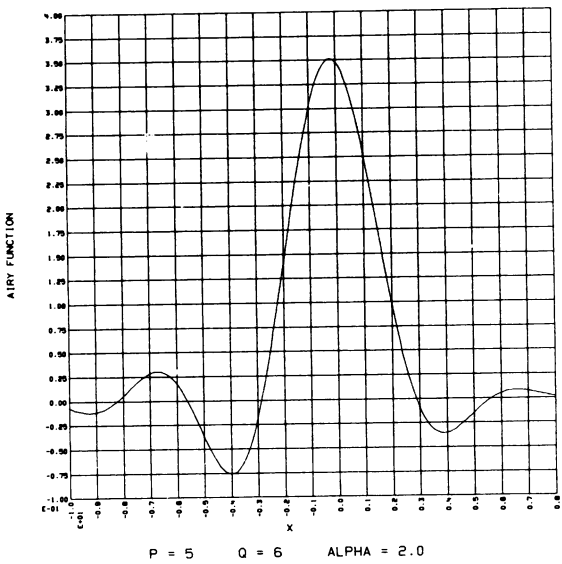


Figure 75

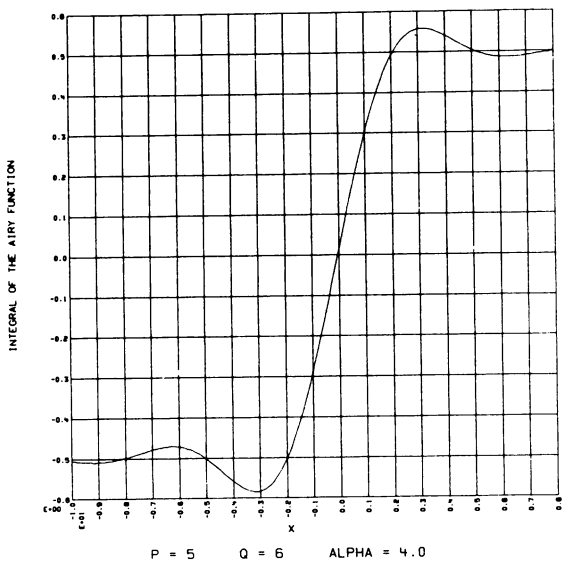


Figure 76

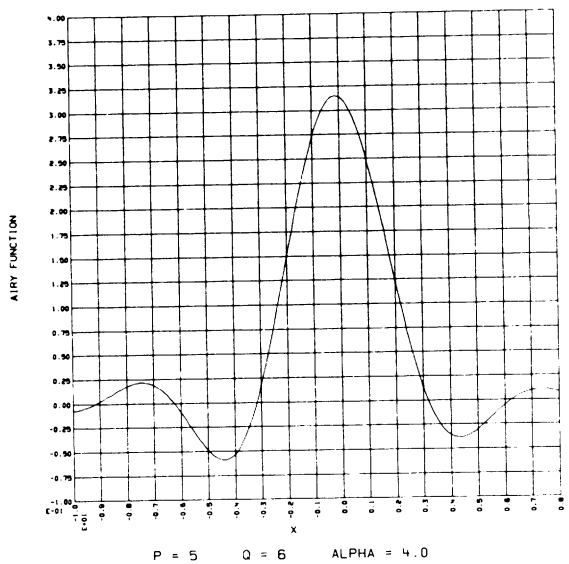


Figure 77



Table 36

P = 5    Q = 6    ALPHA = 0.5

X	ALPHA	ALPHA	X	ALPHA	ALPHA	X	ALPHA	ALPHA	X	ALPHA	ALPHA	X	ALPHA	ALPHA	X	ALPHA	ALPHA	X	ALPHA	ALPHA
-10.0	-0.99999	0.24178	-9.5	-0.99967	0.24089	-1.0	-0.99929	0.23970	-9.5	-0.99887	0.23831	-1.0	-0.99845	0.23682	-9.5	-0.99803	0.23523	-1.0	-0.99761	0.23364
-9.0	-0.99854	0.24422	-8.5	-0.99813	0.24333	-9.0	-0.99771	0.24214	-8.5	-0.99729	0.24075	-9.0	-0.99687	0.23936	-8.5	-0.99645	0.23797	-9.0	-0.99603	0.23658
-8.0	-0.99710	0.24766	-7.5	-0.99669	0.24677	-8.0	-0.99627	0.24558	-7.5	-0.99585	0.24419	-8.0	-0.99543	0.24280	-7.5	-0.99501	0.24141	-8.0	-0.99459	0.24002
-7.0	-0.99566	0.25110	-6.5	-0.99525	0.25021	-7.0	-0.99483	0.24902	-6.5	-0.99441	0.24763	-7.0	-0.99399	0.24624	-6.5	-0.99357	0.24485	-7.0	-0.99315	0.24346
-6.0	-0.99422	0.25454	-5.5	-0.99381	0.25365	-6.0	-0.99339	0.25246	-5.5	-0.99297	0.25107	-6.0	-0.99255	0.24968	-5.5	-0.99213	0.24829	-6.0	-0.99171	0.24690
-5.0	-0.99278	0.25798	-4.5	-0.99237	0.25709	-5.0	-0.99195	0.25590	-4.5	-0.99153	0.25451	-5.0	-0.99111	0.25312	-4.5	-0.99069	0.25173	-5.0	-0.99027	0.25034
-4.0	-0.99134	0.26142	-3.5	-0.99093	0.26053	-4.0	-0.99051	0.25934	-3.5	-0.99009	0.25795	-4.0	-0.98967	0.25656	-3.5	-0.98925	0.25517	-4.0	-0.98883	0.25378
-3.0	-0.99030	0.26486	-2.5	-0.98989	0.26397	-3.0	-0.98947	0.26278	-2.5	-0.98905	0.26139	-3.0	-0.98863	0.26000	-2.5	-0.98821	0.25861	-3.0	-0.98779	0.25722
-2.0	-0.98926	0.26830	-1.5	-0.98885	0.26741	-2.0	-0.98843	0.26622	-1.5	-0.98801	0.26483	-2.0	-0.98759	0.26344	-1.5	-0.98717	0.26205	-2.0	-0.98675	0.26066
-1.0	-0.98822	0.27174	-0.5	-0.98781	0.27085	-1.0	-0.98739	0.26966	-0.5	-0.98697	0.26827	-1.0	-0.98655	0.26688	-0.5	-0.98613	0.26549	-1.0	-0.98571	0.26410
0.0	-0.98718	0.27518	0.5	-0.98677	0.27429	1.0	-0.98635	0.27310	0.5	-0.98593	0.27171	1.0	-0.98551	0.27032	0.5	-0.98509	0.26893	1.0	-0.98467	0.26754
1.0	-0.98614	0.27862	1.5	-0.98573	0.27773	2.0	-0.98531	0.27654	1.5	-0.98489	0.27515	2.0	-0.98447	0.27376	1.5	-0.98405	0.27237	2.0	-0.98363	0.27098
2.0	-0.98510	0.28206	2.5	-0.98469	0.28117	3.0	-0.98427	0.28008	2.5	-0.98385	0.27869	3.0	-0.98343	0.27730	2.5	-0.98301	0.27591	3.0	-0.98259	0.27452
3.0	-0.98406	0.28550	3.5	-0.98365	0.28461	4.0	-0.98323	0.28342	3.5	-0.98281	0.28203	4.0	-0.98239	0.28064	3.5	-0.98197	0.27925	4.0	-0.98155	0.27786
4.0	-0.98302	0.28894	4.5	-0.98261	0.28805	5.0	-0.98219	0.28686	4.5	-0.98177	0.28547	5.0	-0.98135	0.28408	4.5	-0.98093	0.28269	5.0	-0.98051	0.28130
5.0	-0.98198	0.29238	5.5	-0.98157	0.29149	6.0	-0.98115	0.29030	5.5	-0.98073	0.28891	6.0	-0.98031	0.28752	5.5	-0.97989	0.28613	6.0	-0.97947	0.28474
6.0	-0.98094	0.29582	6.5	-0.98053	0.29493	7.0	-0.98011	0.29374	6.5	-0.97969	0.29235	7.0	-0.97927	0.29096	6.5	-0.97885	0.28957	7.0	-0.97843	0.28818
7.0	-0.98000	0.29926	7.5	-0.97959	0.29837	8.0	-0.97917	0.29718	7.5	-0.97875	0.29579	8.0	-0.97833	0.29440	7.5	-0.97791	0.29301	8.0	-0.97749	0.29162
8.0	-0.97906	0.30270	8.5	-0.97865	0.30181	9.0	-0.97823	0.30062	8.5	-0.97781	0.29923	9.0	-0.97739	0.29784	8.5	-0.97697	0.29645	9.0	-0.97655	0.29506
9.0	-0.97812	0.30614	9.5	-0.97771	0.30525	10.0	-0.97729	0.30406	9.5	-0.97687	0.30267	10.0	-0.97645	0.30128	9.5	-0.97603	0.29989	10.0	-0.97561	0.29850
10.0	-0.97718	0.30958	10.5	-0.97677	0.30869	11.0	-0.97635	0.30750	10.5	-0.97593	0.30611	11.0	-0.97551	0.30472	10.5	-0.97509	0.30333	11.0	-0.97467	0.30194







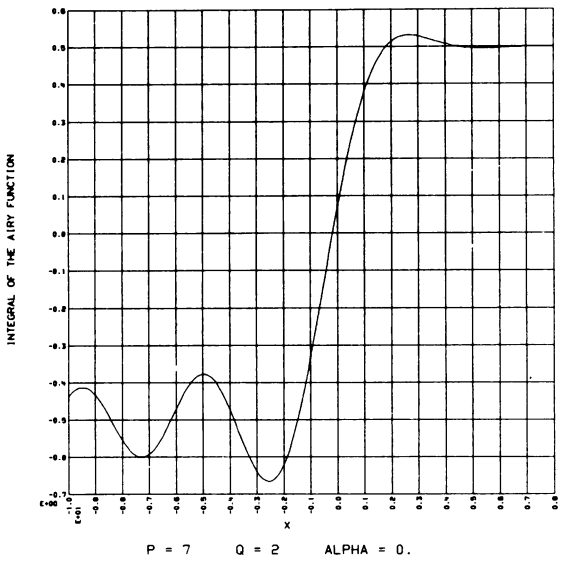


Figure 78

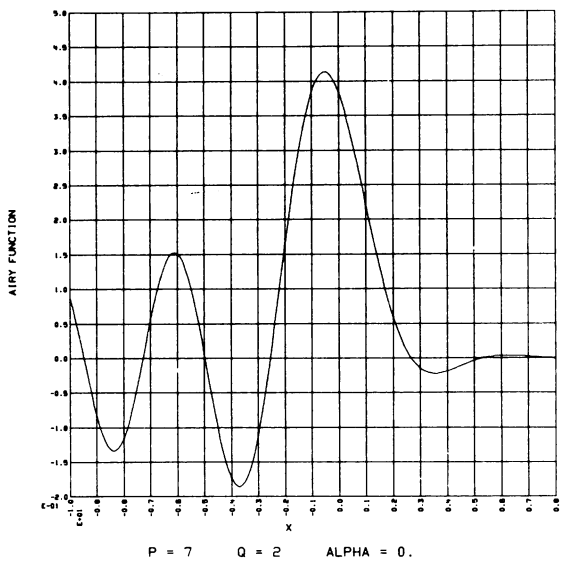


Figure 79

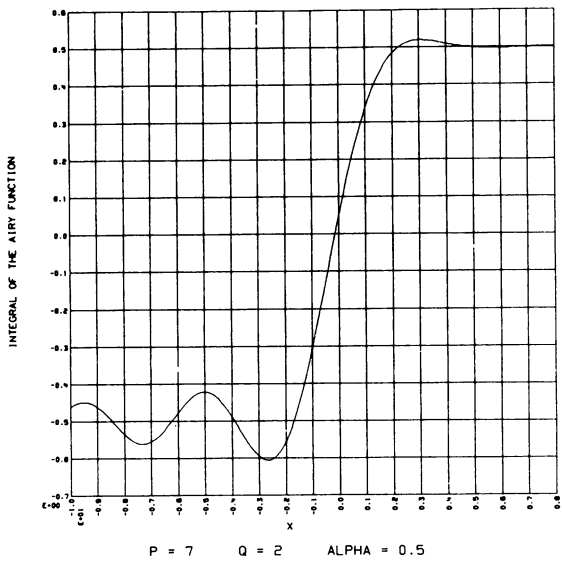


Figure 80

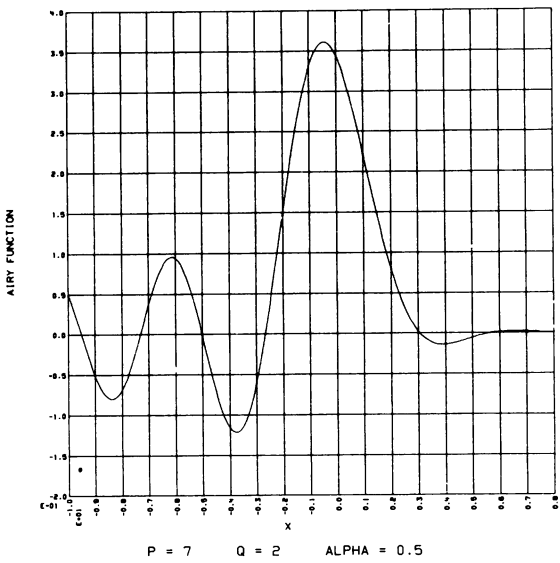


Figure 81



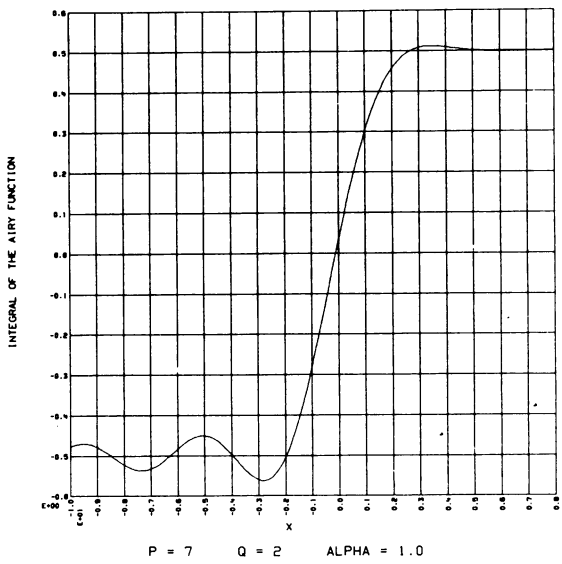


Figure 82

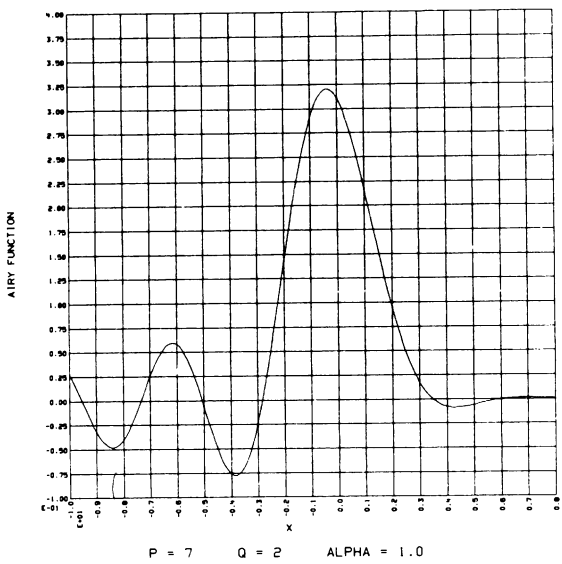


Figure 83

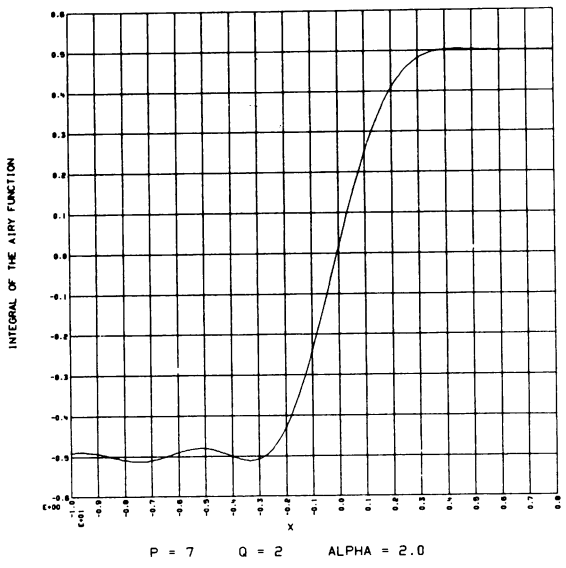


Figure 84

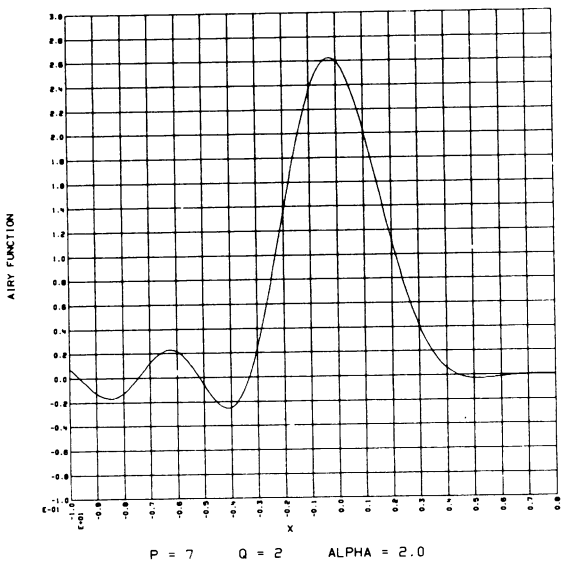


Figure 85

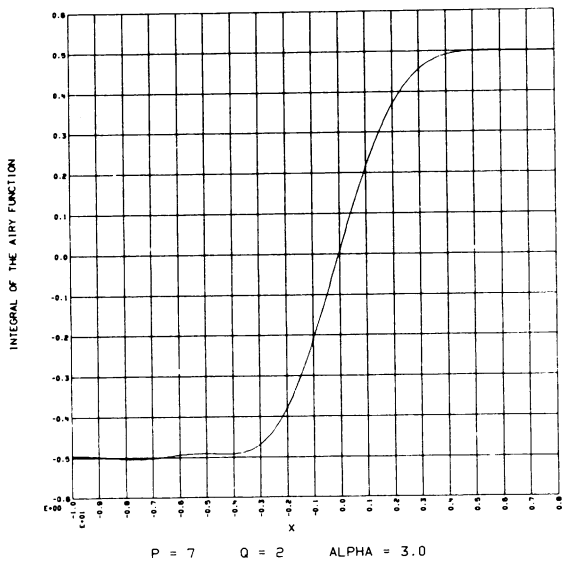


Figure 86

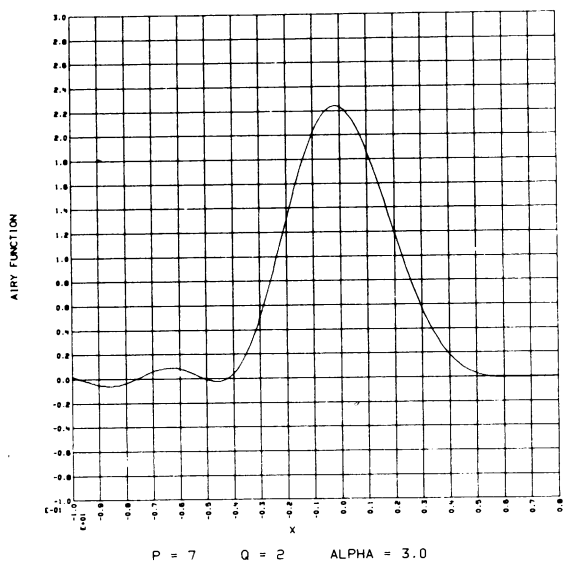


Figure 87

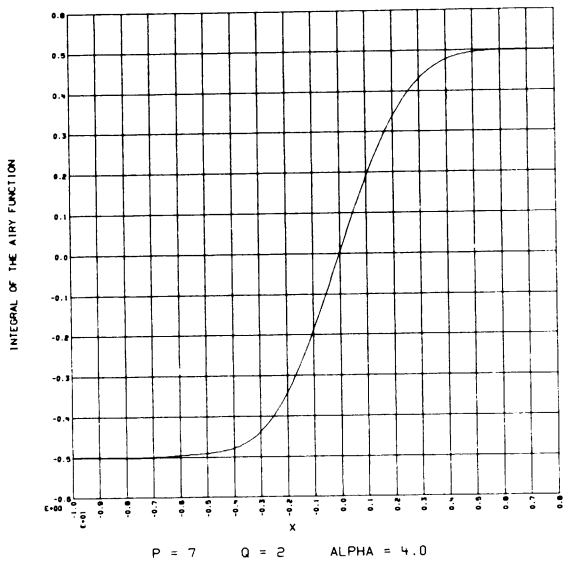


Figure 88

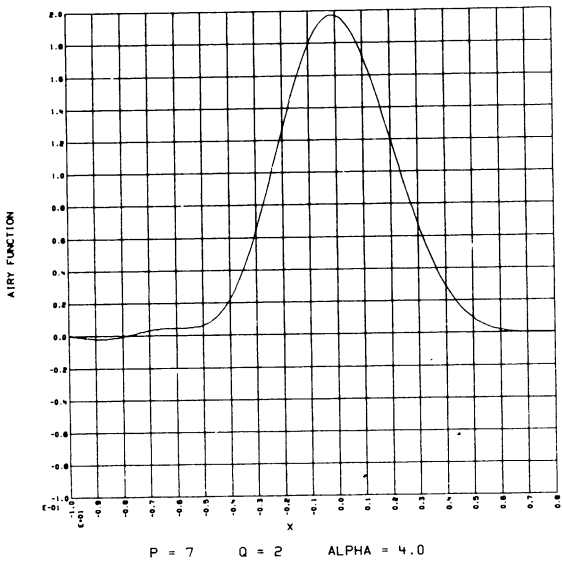


Figure 89













Table 45

P-7 Q-2 ALPHA-N-C

#	ALICE	ALICE	X	ALICE	ALICE	X	ALICE	ALICE	X	ALICE	ALICE	X	ALICE	ALICE			
10	C	0.00000	C	0.00000	-5.5	C	0.03070	0.02730	-1.0	C	0.18770	0.17680	5.3	C	0.00000	C	0.00000
9	C	0.00070	C	0.00050	-5.1	C	0.03001	0.02690	-0.9	C	0.09663	0.08520	5.6	C	0.00000	C	0.00000
-9	C	0.00091	C	0.00074	-5.1	C	0.03007	0.02697	-0.9	C	0.01074	0.00970	5.7	C	0.00000	C	0.00000
-9	C	0.00099	C	0.00082	-5.1	C	0.03012	0.02702	-0.7	C	0.03001	0.02810	5.8	C	0.00000	C	0.00000
-9	C	0.00101	C	0.00084	-5.1	C	0.03017	0.02707	-0.5	C	0.03001	0.02810	5.9	C	0.00000	C	0.00000
-9	C	0.00102	C	0.00085	-5.0	C	0.03022	0.02712	-0.5	C	0.03001	0.02810	6.0	C	0.00000	C	0.00000
-9	C	0.00103	C	0.00086	-5.0	C	0.03027	0.02717	-0.4	C	0.03001	0.02810	6.1	C	0.00000	C	0.00000
-9	C	0.00104	C	0.00087	-5.0	C	0.03032	0.02722	-0.3	C	0.03001	0.02810	6.2	C	0.00000	C	0.00000
-9	C	0.00105	C	0.00088	-5.0	C	0.03037	0.02727	-0.2	C	0.03001	0.02810	6.3	C	0.00000	C	0.00000
-9	C	0.00106	C	0.00089	-5.0	C	0.03042	0.02732	-0.1	C	0.03001	0.02810	6.4	C	0.00000	C	0.00000
-9	C	0.00107	C	0.00090	-5.0	C	0.03047	0.02737	0.0	C	0.03001	0.02810	6.5	C	0.00000	C	0.00000
-9	C	0.00108	C	0.00091	-5.0	C	0.03052	0.02742	0.1	C	0.03001	0.02810	6.6	C	0.00000	C	0.00000
-9	C	0.00109	C	0.00092	-5.0	C	0.03057	0.02747	0.2	C	0.03001	0.02810	6.7	C	0.00000	C	0.00000
-9	C	0.00110	C	0.00093	-5.0	C	0.03062	0.02752	0.3	C	0.03001	0.02810	6.8	C	0.00000	C	0.00000
-9	C	0.00111	C	0.00094	-5.0	C	0.03067	0.02757	0.4	C	0.03001	0.02810	6.9	C	0.00000	C	0.00000
-9	C	0.00112	C	0.00095	-5.0	C	0.03072	0.02762	0.5	C	0.03001	0.02810	7.0	C	0.00000	C	0.00000
-9	C	0.00113	C	0.00096	-5.0	C	0.03077	0.02767	0.6	C	0.03001	0.02810	7.1	C	0.00000	C	0.00000
-9	C	0.00114	C	0.00097	-5.0	C	0.03082	0.02772	0.7	C	0.03001	0.02810	7.2	C	0.00000	C	0.00000
-9	C	0.00115	C	0.00098	-5.0	C	0.03087	0.02777	0.8	C	0.03001	0.02810	7.3	C	0.00000	C	0.00000
-9	C	0.00116	C	0.00099	-5.0	C	0.03092	0.02782	0.9	C	0.03001	0.02810	7.4	C	0.00000	C	0.00000
-9	C	0.00117	C	0.00100	-5.0	C	0.03097	0.02787	1.0	C	0.03001	0.02810	7.5	C	0.00000	C	0.00000
-9	C	0.00118	C	0.00101	-5.0	C	0.03102	0.02792	1.1	C	0.03001	0.02810	7.6	C	0.00000	C	0.00000
-9	C	0.00119	C	0.00102	-5.0	C	0.03107	0.02797	1.2	C	0.03001	0.02810	7.7	C	0.00000	C	0.00000
-9	C	0.00120	C	0.00103	-5.0	C	0.03112	0.02802	1.3	C	0.03001	0.02810	7.8	C	0.00000	C	0.00000
-9	C	0.00121	C	0.00104	-5.0	C	0.03117	0.02807	1.4	C	0.03001	0.02810	7.9	C	0.00000	C	0.00000
-9	C	0.00122	C	0.00105	-5.0	C	0.03122	0.02812	1.5	C	0.03001	0.02810	8.0	C	0.00000	C	0.00000

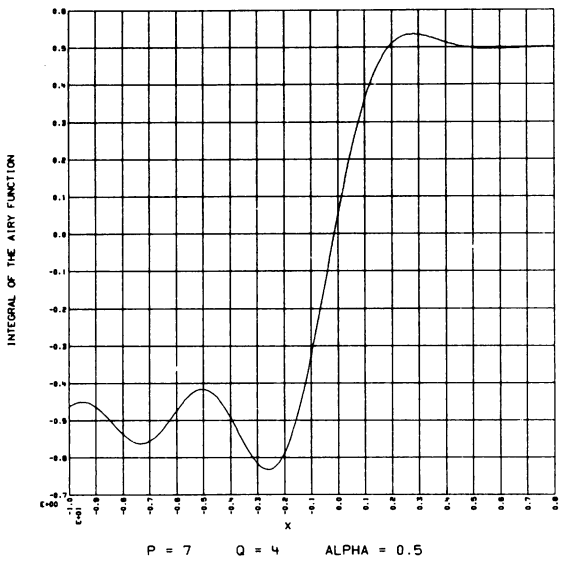


Figure 90

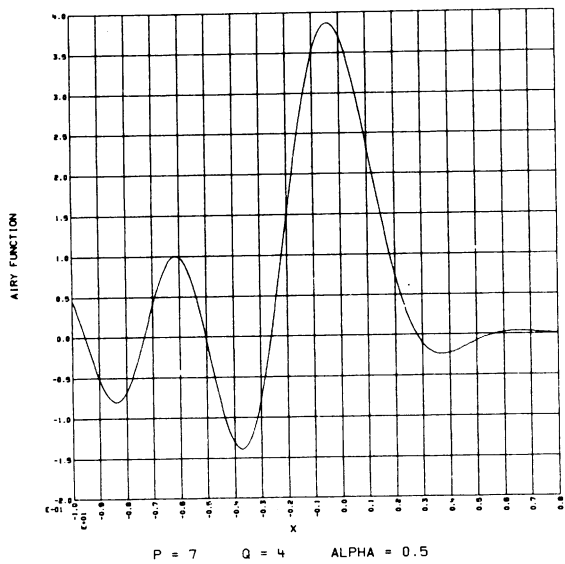


Figure 91



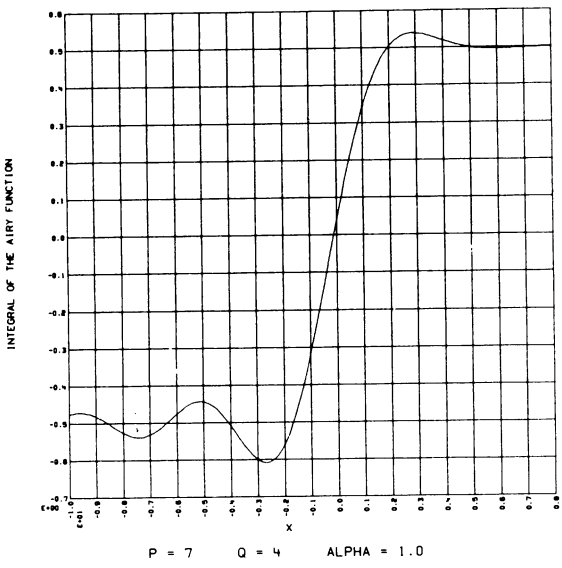


Figure 92

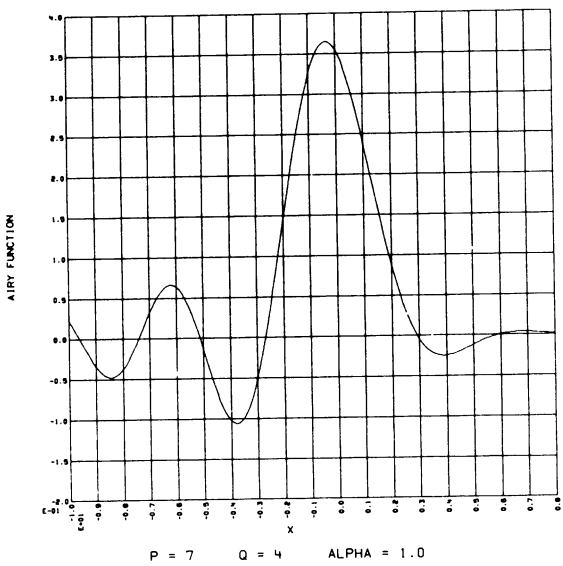


Figure 93

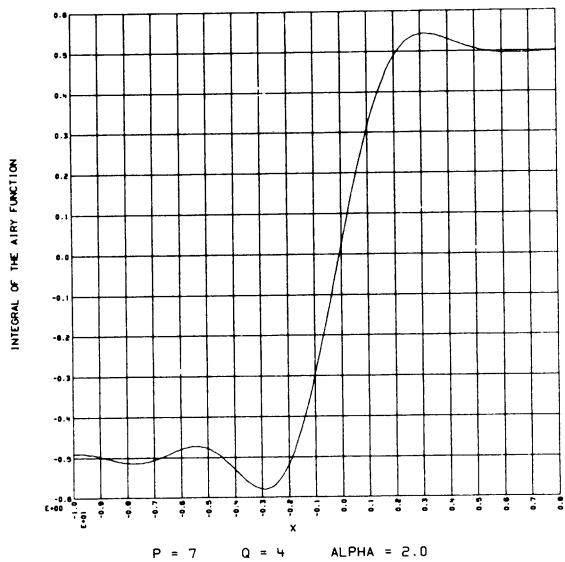


Figure 94

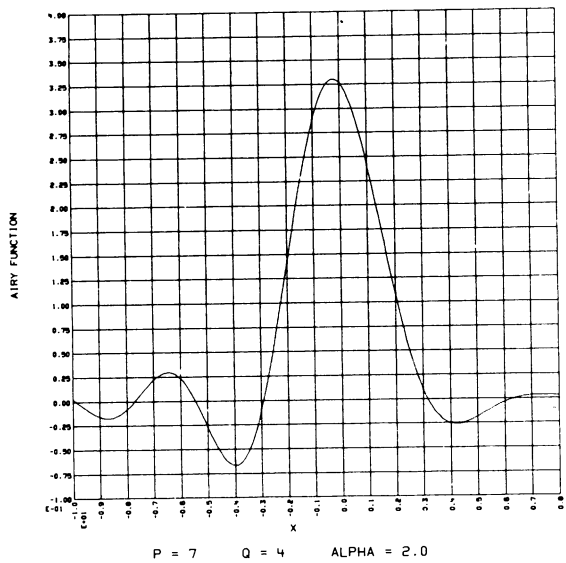


Figure 95

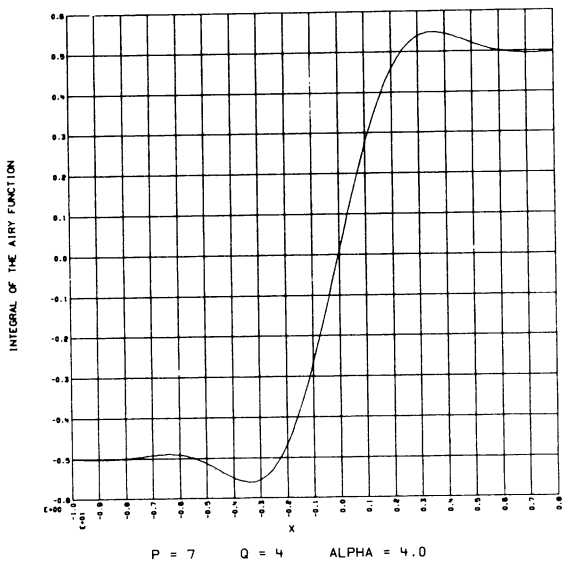


Figure 96

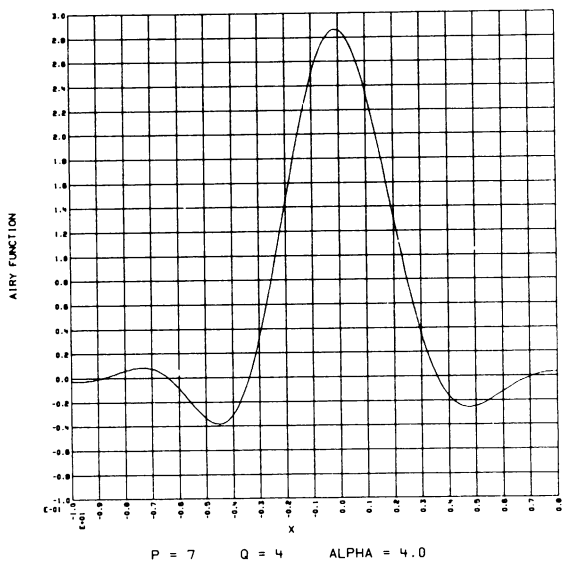


Figure 97







Table 48

P T Q -- ALPHA - Z C

X	ATL1K1	ATL1K1	X	ATL1K1	ATL1K1	X	ATL1K1	ATL1K1	X	ATL1K1	ATL1K1	X	ATL1K1	ATL1K1
-10	-0.90381	0.002351	-5.5	-0.472876	0.002447	-1.0	-0.296501	0.799664	3.5	0.545274	-0.015103	6.0	-0.90381	0.002351
-9	-0.90381	0.002351	-5.4	-0.472876	0.002351	-0.9	-0.296501	0.799664	3.6	0.545274	-0.015103	5.9	-0.90381	0.002351
-8	-0.90381	0.002351	-5.3	-0.472876	0.002351	-0.8	-0.296501	0.799664	3.7	0.545274	-0.015103	5.8	-0.90381	0.002351
-7	-0.90381	0.002351	-5.2	-0.472876	0.002351	-0.7	-0.296501	0.799664	3.8	0.545274	-0.015103	5.7	-0.90381	0.002351
-6	-0.90381	0.002351	-5.1	-0.472876	0.002351	-0.6	-0.296501	0.799664	3.9	0.545274	-0.015103	5.6	-0.90381	0.002351
-5	-0.90381	0.002351	-5.0	-0.472876	0.002351	-0.5	-0.296501	0.799664	4.0	0.545274	-0.015103	5.5	-0.90381	0.002351
-4	-0.90381	0.002351	-4.9	-0.472876	0.002351	-0.4	-0.296501	0.799664	4.1	0.545274	-0.015103	5.4	-0.90381	0.002351
-3	-0.90381	0.002351	-4.8	-0.472876	0.002351	-0.3	-0.296501	0.799664	4.2	0.545274	-0.015103	5.3	-0.90381	0.002351
-2	-0.90381	0.002351	-4.7	-0.472876	0.002351	-0.2	-0.296501	0.799664	4.3	0.545274	-0.015103	5.2	-0.90381	0.002351
-1	-0.90381	0.002351	-4.6	-0.472876	0.002351	-0.1	-0.296501	0.799664	4.4	0.545274	-0.015103	5.1	-0.90381	0.002351
0	-0.90381	0.002351	-4.5	-0.472876	0.002351	0.0	-0.296501	0.799664	4.5	0.545274	-0.015103	5.0	-0.90381	0.002351
1	-0.90381	0.002351	-4.4	-0.472876	0.002351	0.1	-0.296501	0.799664	4.6	0.545274	-0.015103	4.9	-0.90381	0.002351
2	-0.90381	0.002351	-4.3	-0.472876	0.002351	0.2	-0.296501	0.799664	4.7	0.545274	-0.015103	4.8	-0.90381	0.002351
3	-0.90381	0.002351	-4.2	-0.472876	0.002351	0.3	-0.296501	0.799664	4.8	0.545274	-0.015103	4.7	-0.90381	0.002351
4	-0.90381	0.002351	-4.1	-0.472876	0.002351	0.4	-0.296501	0.799664	4.9	0.545274	-0.015103	4.6	-0.90381	0.002351
5	-0.90381	0.002351	-4.0	-0.472876	0.002351	0.5	-0.296501	0.799664	5.0	0.545274	-0.015103	4.5	-0.90381	0.002351
6	-0.90381	0.002351	-3.9	-0.472876	0.002351	0.6	-0.296501	0.799664	5.1	0.545274	-0.015103	4.4	-0.90381	0.002351
7	-0.90381	0.002351	-3.8	-0.472876	0.002351	0.7	-0.296501	0.799664	5.2	0.545274	-0.015103	4.3	-0.90381	0.002351
8	-0.90381	0.002351	-3.7	-0.472876	0.002351	0.8	-0.296501	0.799664	5.3	0.545274	-0.015103	4.2	-0.90381	0.002351
9	-0.90381	0.002351	-3.6	-0.472876	0.002351	0.9	-0.296501	0.799664	5.4	0.545274	-0.015103	4.1	-0.90381	0.002351
10	-0.90381	0.002351	-3.5	-0.472876	0.002351	1.0	-0.296501	0.799664	5.5	0.545274	-0.015103	4.0	-0.90381	0.002351



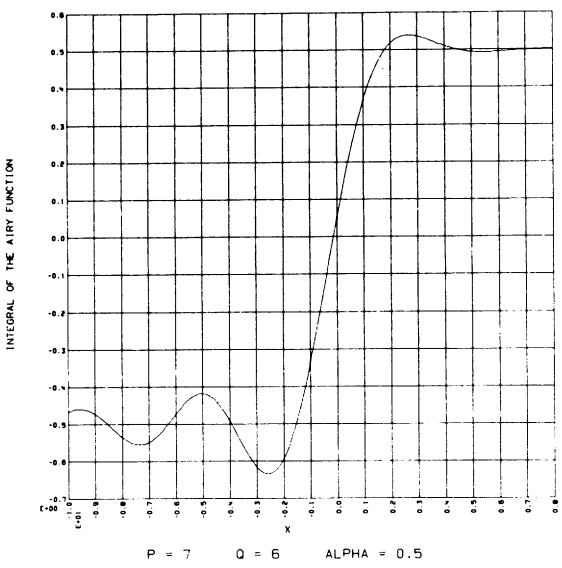


Figure 98

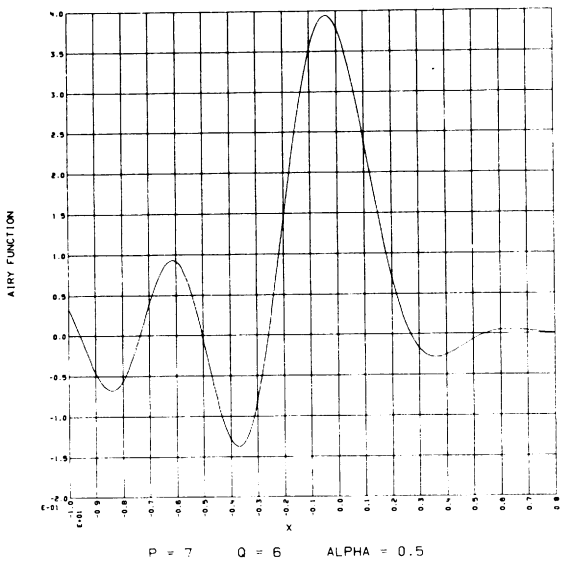


Figure 99

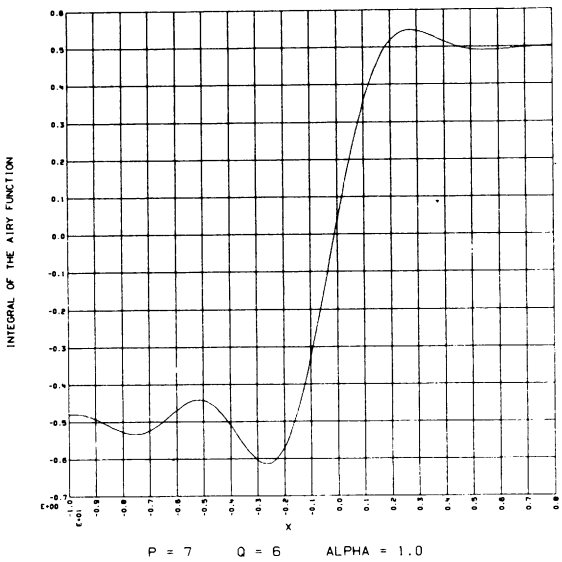


Figure 100



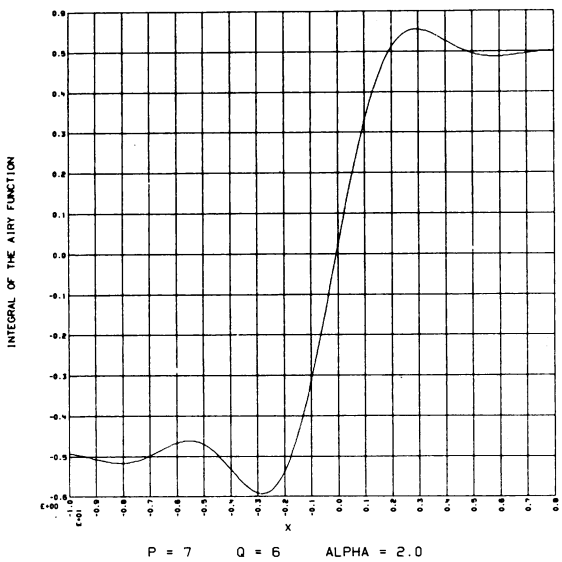
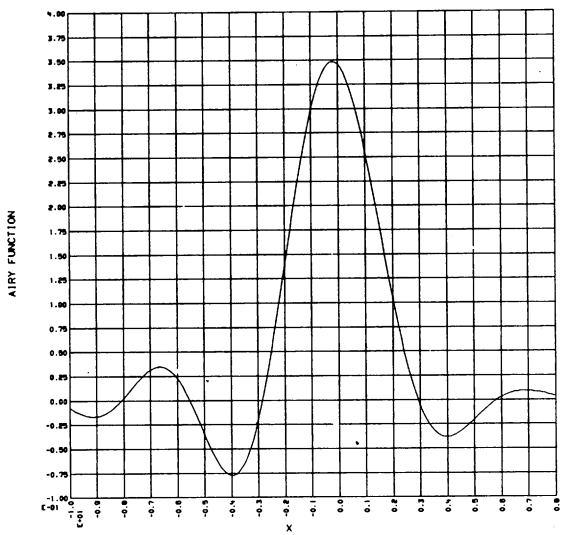


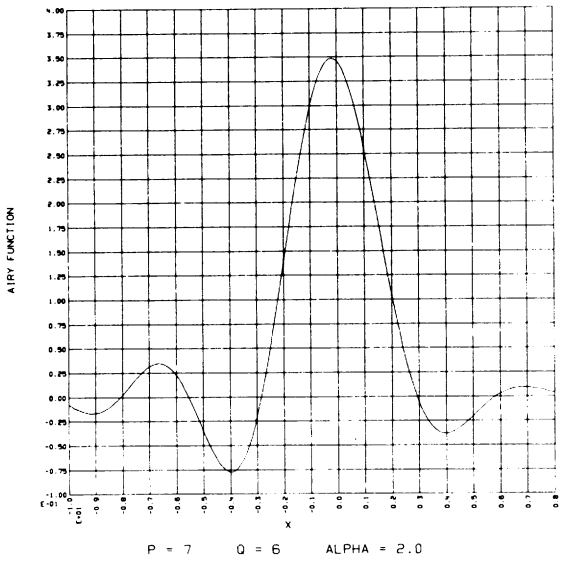
Figure 102



P = 7    Q = 6    ALPHA = 2.0

Figure 103





P = 7      Q = 6      ALPHA = 2.0

Figure 103

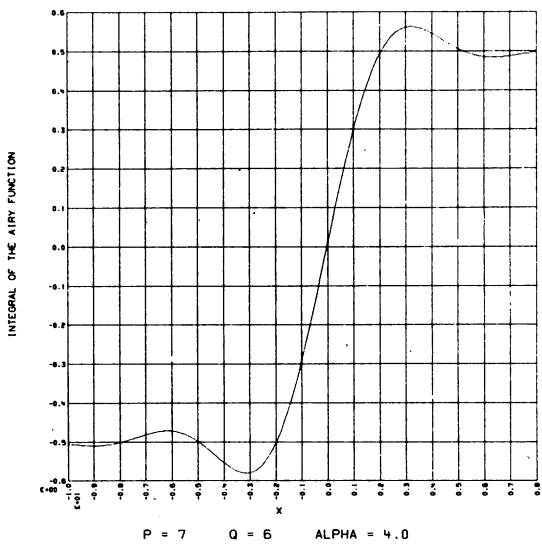


Figure 104

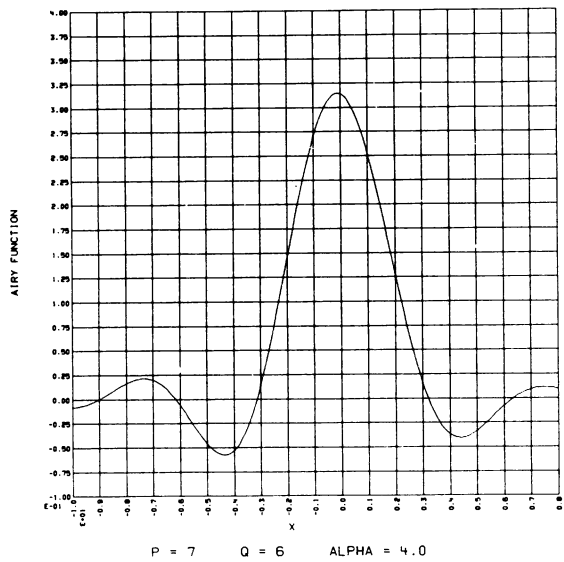


Figure 105





